Horizontal and Vertical Translations

Translations

A __translation__ alters a graph by altering its __position__, __shape__, and/or __orientation__

A __translation__ is a type of transformation that alters the position of a graph. The shape and orientation do not change.

Example 1

Triangle ABC has undergone a translation 5 units to the right and 1 unit down.

We can express this translation in __mapping notation__ as follows.

ORIGINAL graph   TRANSLATED (or IMAGE) graph

$$(x,y) \longrightarrow (x+5, y-1)$$

ORIGINAL POINTS	TRANSLATED POINTS
A $(-4,5)$	$(-4,5) \longrightarrow (-4+5, 5-1) = (1,4) = A'$
B $(-3,2)$	$(2,1)$
C $(-1,3)$	$(4,2)$
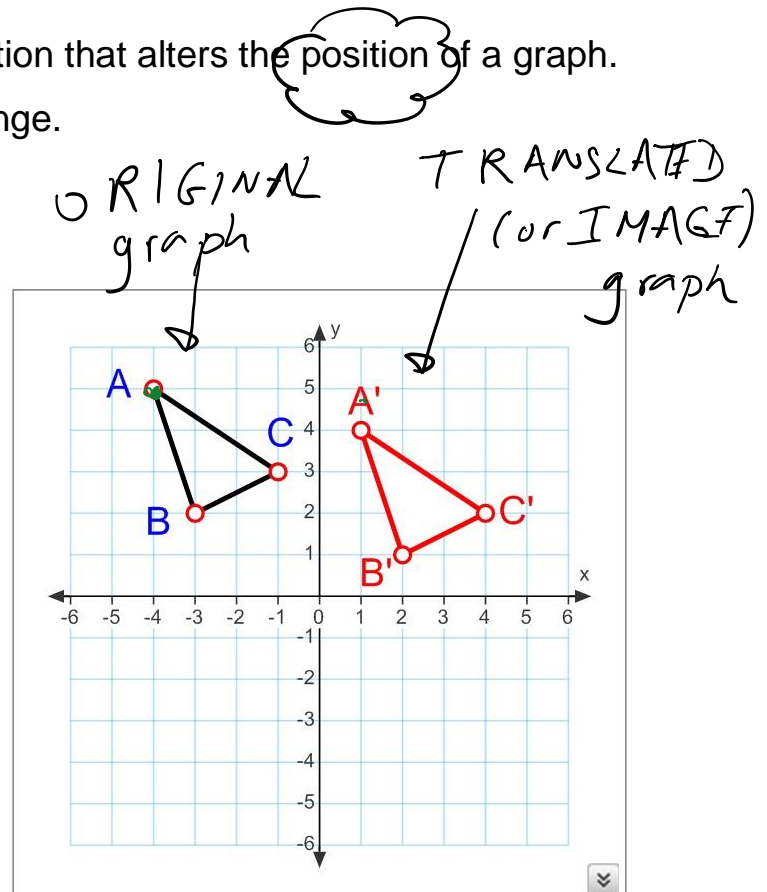
✳ If a point moves:
- RIGHT/LEFT : x-coordinate changes
- UP/Down : y-coordinate changes

Practice Questions

Graph the image of the figure using the transformation given.

a) graph the image   b) write the mapping

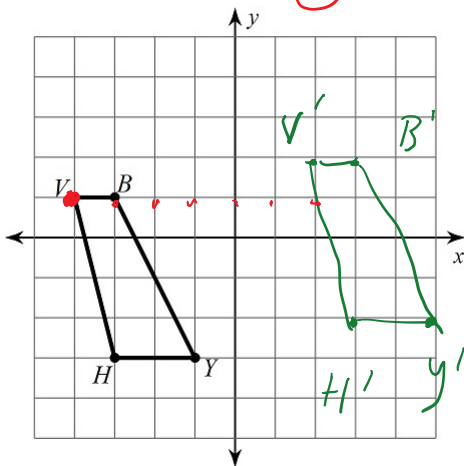$\longrightarrow x+2$ $\uparrow y+1$

translation: 2 units right and 1 unit up
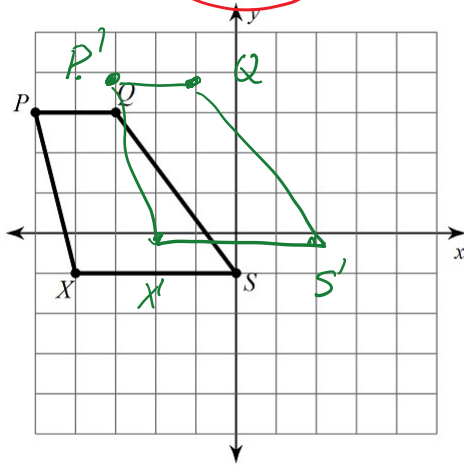
$(x, y) \rightarrow (x+2, y+1)$

c) Graph the image of the figure using the transformation given.
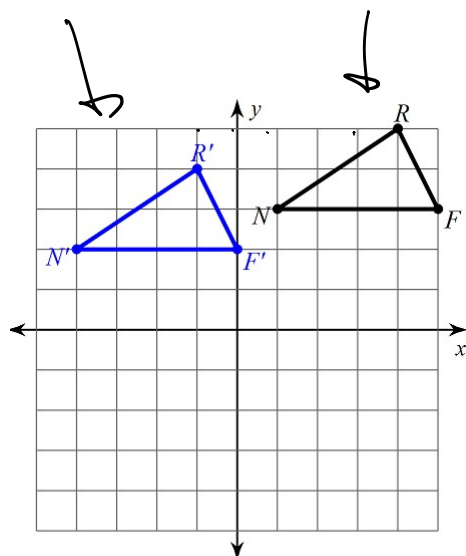
translation: $(x, y) \rightarrow (x+6, y+1)$

$\longrightarrow x+6$ $\uparrow y+1$

Describe a rule for the following translation using mapping notation.
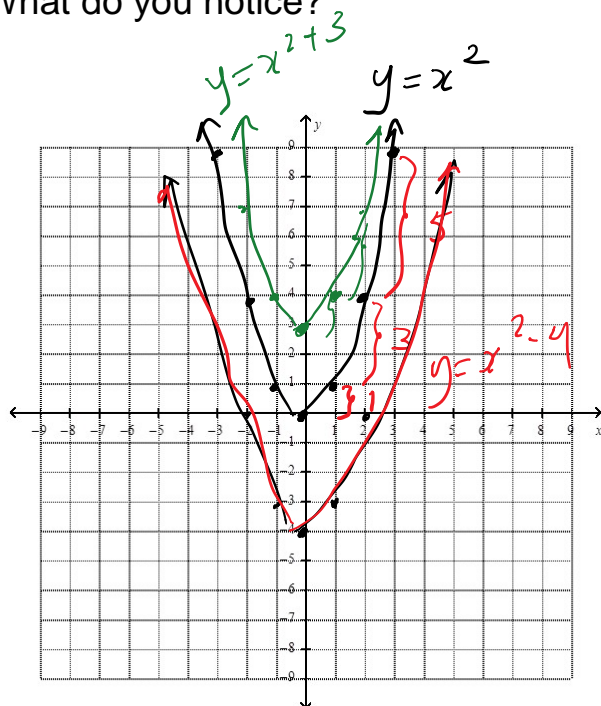
TRANSLATED   ORIGINAL

mapping notation:

$$(x, y) \rightarrow (x - 5, y - 1)$$

Vertical Shifts (Translations)

Graph the function $y = x^2$, $y = x^2 + 3$, $y = x^2 - 4$ on the same graph.

What do you notice?

$y = x^2 + 3$ $y = x^2$

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = x^2 + 3$

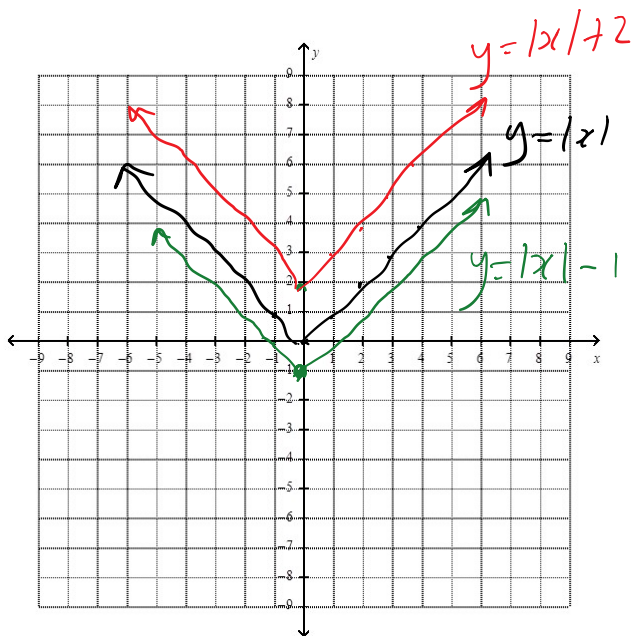| x	y
-2	7
-1	4
0	3
1	4
2	7

$y = x^2 - 4$

x	y
-2	0
-1	-3
0	-4
1	-3
2	0

Graph the function $y = |x|$, $y = |x| - 1$, $y = |x| + 2$ on the same graph.

What do you notice?

$y = |x|$ means $\begin{cases} y = x & \text{if } x \geq 0 \\ y = -x & \text{if } x < 0 \end{cases}$

$$y = |x|$$ $$y = |x| - 1$$

x	y
-2	2
-1	1
0	0
1	1
2	2

x	y
-2	1
-1	0
0	-1
1	0
2	1

$y = |x| + 2$

$y = |x|$

$y = |x| - 1$
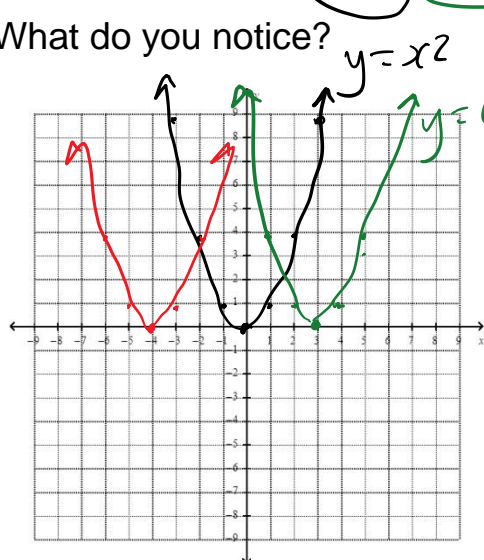
$$y = |x| + 2$$

x	y
-2	
-1	
0	
1	

In general $y - k = f(x)$ or $y = f(x) + k$ represents a vertical translation of

the graph of the function $y = f(x)$. If $k > 0$, then the graph is translated

k units up. If $k < 0$, then the graph is translated $|k|$ units down.

vertical
translation

$+k$ $+k$

Horizontal Shifts (Translations)

vertex form of parabola : $y = a(x - h)^2 + k$ $k +$ $k -$

Graph the function $y = x^2$, $y = (x - 3)^2$, $y = (x + 4)^2$ on the same graph.
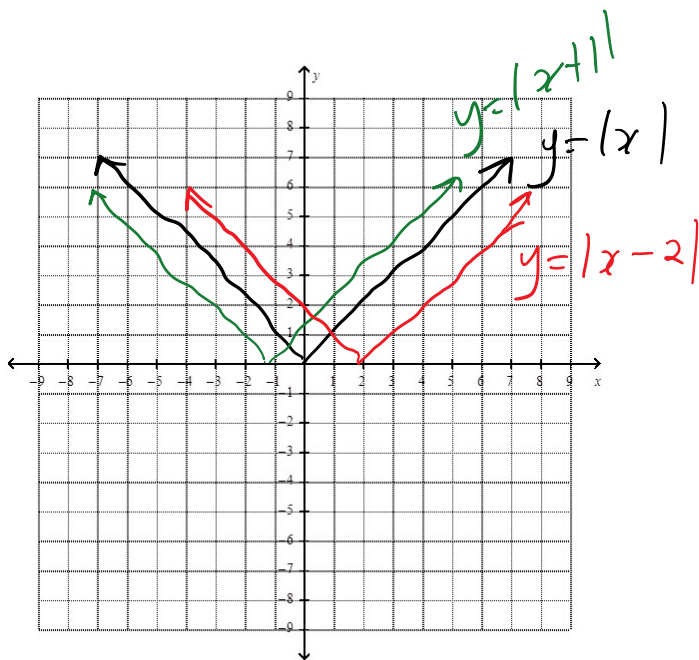
What do you notice?

$y = x^2$ $y = (x - 3)^2$ $y = (x + 4)^2$

x	y
1	$(-2)^2 = 4$
2	1
3	0
4	$(1)^2 = 1$
5	$(2)^2 = 4$

x	y
-6	4
-5	1
-4	0
-3	1
-2	4

Graph the function $y = |x|$, $y = |x + 1|$, $y = |x - 2|$ on the same graph.

What do you notice?

$y = |x+1|$ $y = |x-2|$

x	y
-2	1
-1	0
0	1
1	2
2	3

In general $y = f(x - h)$ represents a horizontal translation of the graph of the function $y = f(x)$. If $h > 0$, then the graph is translated h units to the right. If $h < 0$, then the graph is translated $|h|$ units to the left.
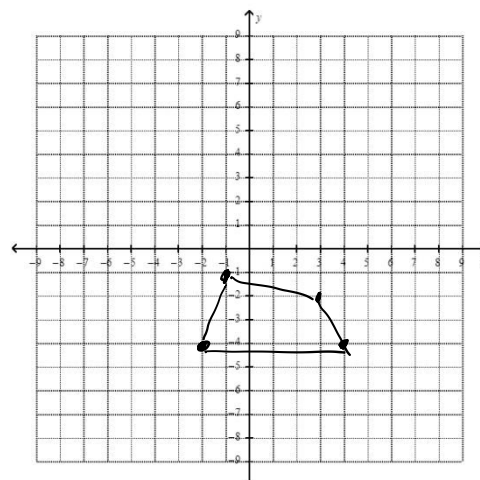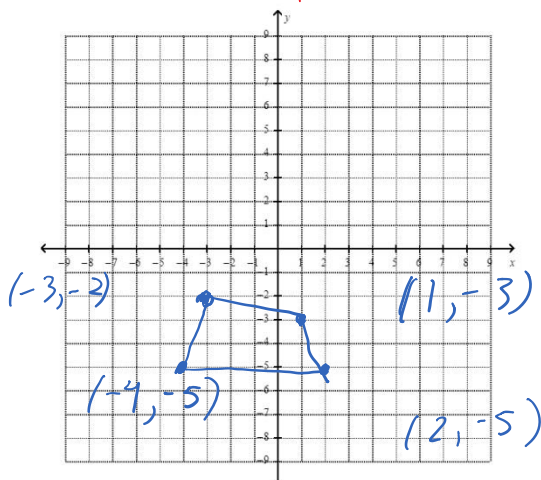
$y = |x+1|$ → $h > 1 \to$ move RIGHT
$= |x - (-1)|$ $h < 1 \to$ move LEFT

Example 2

Given the graph of $y = f(x)$ sketch the graph of the transformed function $y = f(x - 2) + 1$.   vertical → up 1

horizontal → RIGHT 2

$(-3, -2)$ $(1, -3)$

$(-4, -5)$

$(2, -5)$

summary: $y = f(x - h) + k$ vertical

↑ horizontal