Reflections and Stretches

A reflection is a transformation that flips an image over a line of reflection. A reflection results in a mirror image of the original shape.

Reflected about x-axis

A (3,5) A' (3,-5)

B (3,3) B' (3,-3)

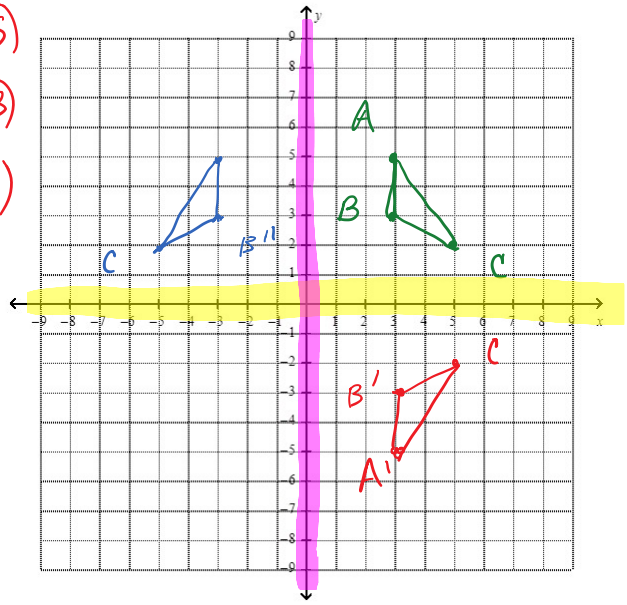
C (5,2) C' (5,-2)

Reflected about y-axis

A'' (-3, 5)

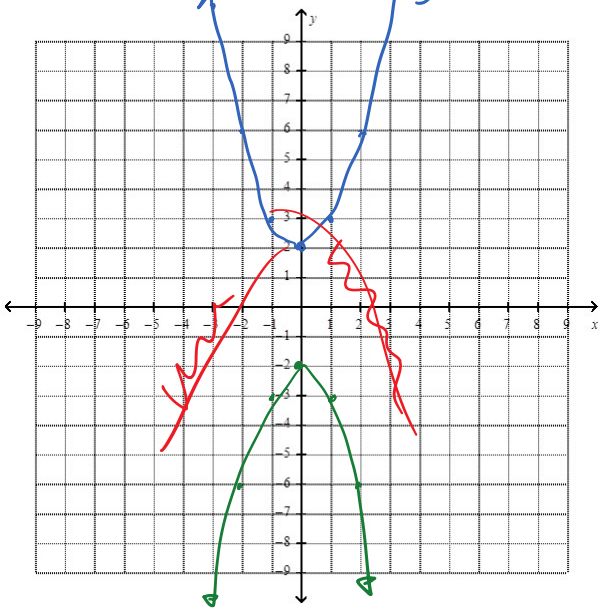
B'' (-3, 3)

C'' (-5, 2)



Draw a graph of $y = x^2 + 2$ and $y = -f(x)$ on the same graph. What do you notice?

reflected over vertical up 2



$y = -f(x)$

$= -(x^2 + 2)$

$y = -x^2 - 2$

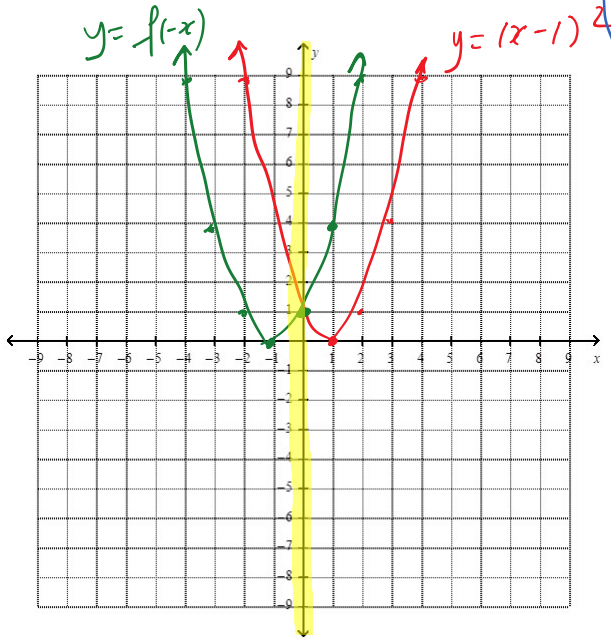
x	y
0	-2
1	-3
2	-6

Draw a graph of $y = (x-1)^2$ and $y = f(-x)$ on the same graph. What do you notice?

horizontal
h = 1, RIGHT 1

$f(-x) = (-x-1)^2$

x	$f(-x)$
-2	1
-1	0
0	1
1	$(-2)^2 = 4$
2	$(-3)^2 = 9$



mapping notation

- $y = f(x)$
- $y = -f(x)$ Reflection in x-axis
  $(x, y) \to (x, -y)$
- $y = f(-x)$ Reflection in y-axis
  $(x, y) \to (-x, y)$

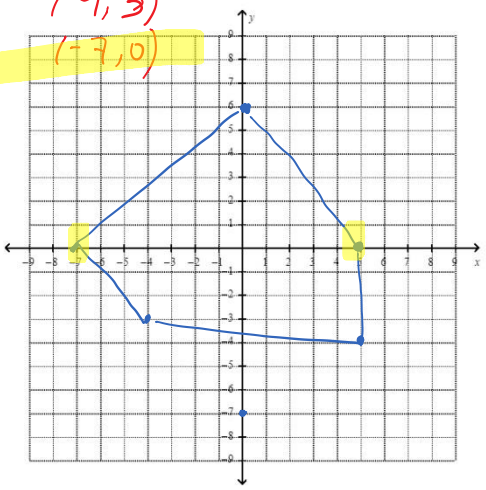
In general $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$ and $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.

An invariant point, is a point on a graph that does not change position after the graph undergoes a transformation.

(0, 6) (5, 0) (5, -4) (-4, -3) (-7, 0)
Reflection in x-axis
(0, -6) (5, 0) (5, 4) (-4, 3) (-7, 0)

Example

Consider the following graph.



Sketch the reflection of this graph in the x-axis. State any invariant points.

Stretches

A STRETCH is a transformation that changes the shape of a graph, but not the orientation.

Plot

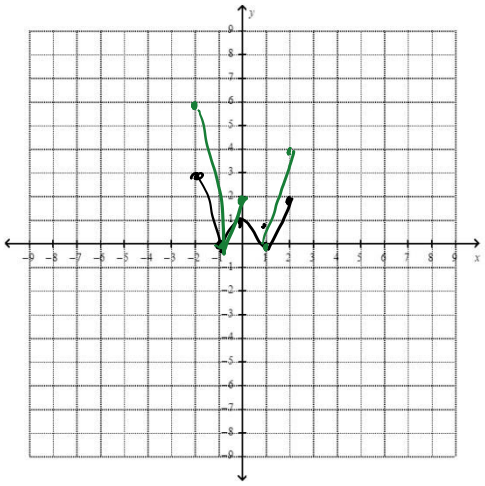
(-2, 3)

(-1, 0)

(0, 1)

(1, 0)

(2, 2)

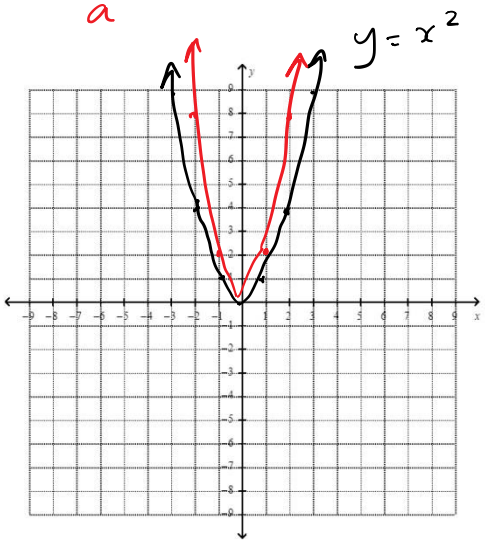


vertical stretch
- taller

Draw a graph of $f(x) = x^2$ and $2f(x)$ on the same graph. What do you notice?

a

x	$y = 2x^2$
-2	$2(-2)^2 = 8$
-1	$2(-1)^2 = 2$
0	0
1	$2(1) = 2$
2	$2(4) = 8$



mapping notation:

$(x, y) \rightarrow (x, 2y)$

This is a vertical stretch by a factor of 2.

$(a = 2)$

- TALLER

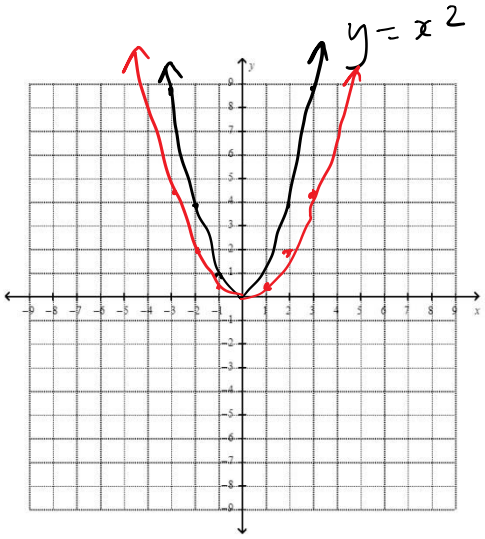
Draw a graph of $f(x) = x^2$ and $\frac{1}{2}f(x)$ on the same graph. What do you notice?

$y = \frac{1}{2} f(x)$ $f(x) = x^2$

$y = \frac{1}{2} f(x)$

$= \frac{1}{2} x^2$

$\frac{1}{2}$ ← a



In general $y = af(x)$ is a vertical stretch about the x-axis of the graph of the function $f(x)$ by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

↳ flipped

$(x, y) \rightarrow (x, ay)$ $a > 1$: taller, $0 < a < 1$: wider

Draw a graph of $f(x) = x^2$ and $f(2x)$ on the same graph. What do you notice?

$y = f(2x)$

$= (2x)^2$

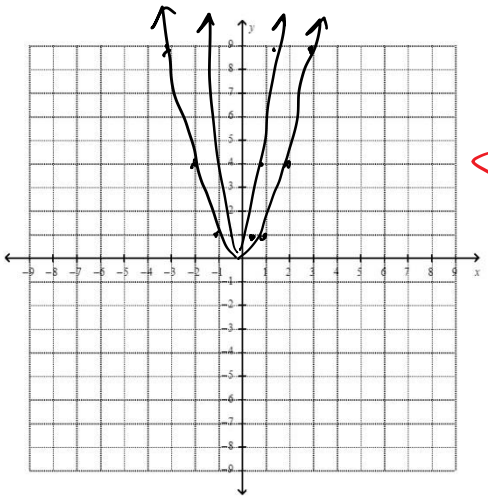
$y = 4x^2$

x	y
$\frac{1}{2}$	1
$-\frac{1}{2}$	1

$1 = 4x^2$

$\frac{1}{4} = x^2$

$x = \pm\frac{1}{2}$



It looks like x-coordinate is being halved.

SQUISHED INWARDS

Narrower

$(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$

Draw a graph of $f(x) = x^2$ and $f\left(\frac{1}{2}x\right)$ on the same graph. What do you notice?

$f\left(\frac{1}{2}x\right)$

$= \left(\frac{1}{2}x\right)^2$

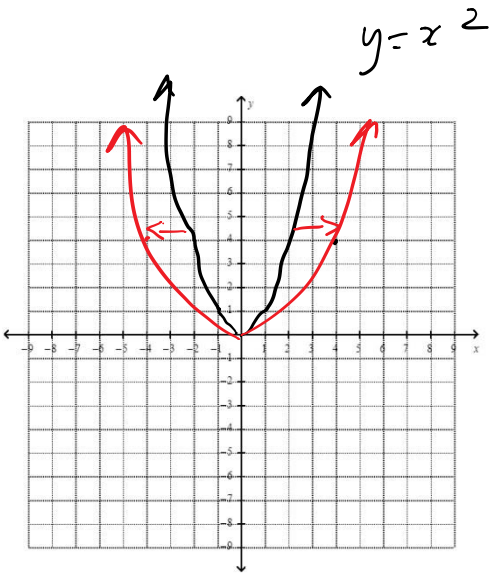
$= \frac{1}{4}x^2$

$4 = \frac{1}{4}x^2$

$4 = \frac{1}{4}x^2$

$x^2 = 16$

$x = \pm 4$



x	y
-2	1
2	1
4	4
-4	4

stretched outwards - WIDER

In general $y = f(bx)$ is a horizontal stretch about the y-axis of the graph of the function $y = f(x)$ by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$f(bx)$

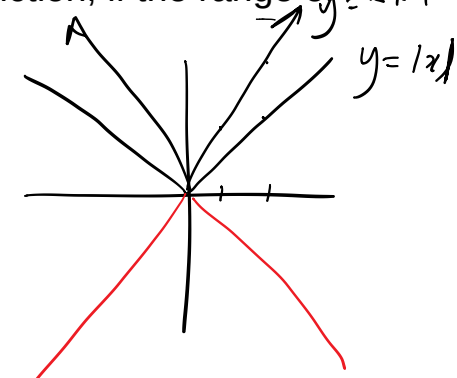
$(x, y) \rightarrow \left(\frac{1}{b}x, y\right)$

Example 1

The graph of the function $y = |x|$ has been stretched vertically about the x-axis by a factor of 2. Write the equation of the transformed function, if the range of the transformed function is $\{y | y \leq 0, y \in R\}$.

$y = af(x)$

$= -2f(x)$



Given the graph of $y = f(x)$, sketch the rpah of the transformed function $y = f(-\frac{1}{2}x)$.

$y = f(bx)$ Horizontal stretch by a factor of $\frac{1}{|b|} = \frac{1}{\left|-\frac{1}{2}\right|} = 2$

$\rightarrow \left(\frac{1}{b}x, y\right)$

$(x,y) \rightarrow (-2x, y)$

$(x,y) \rightarrow (2x, y)$

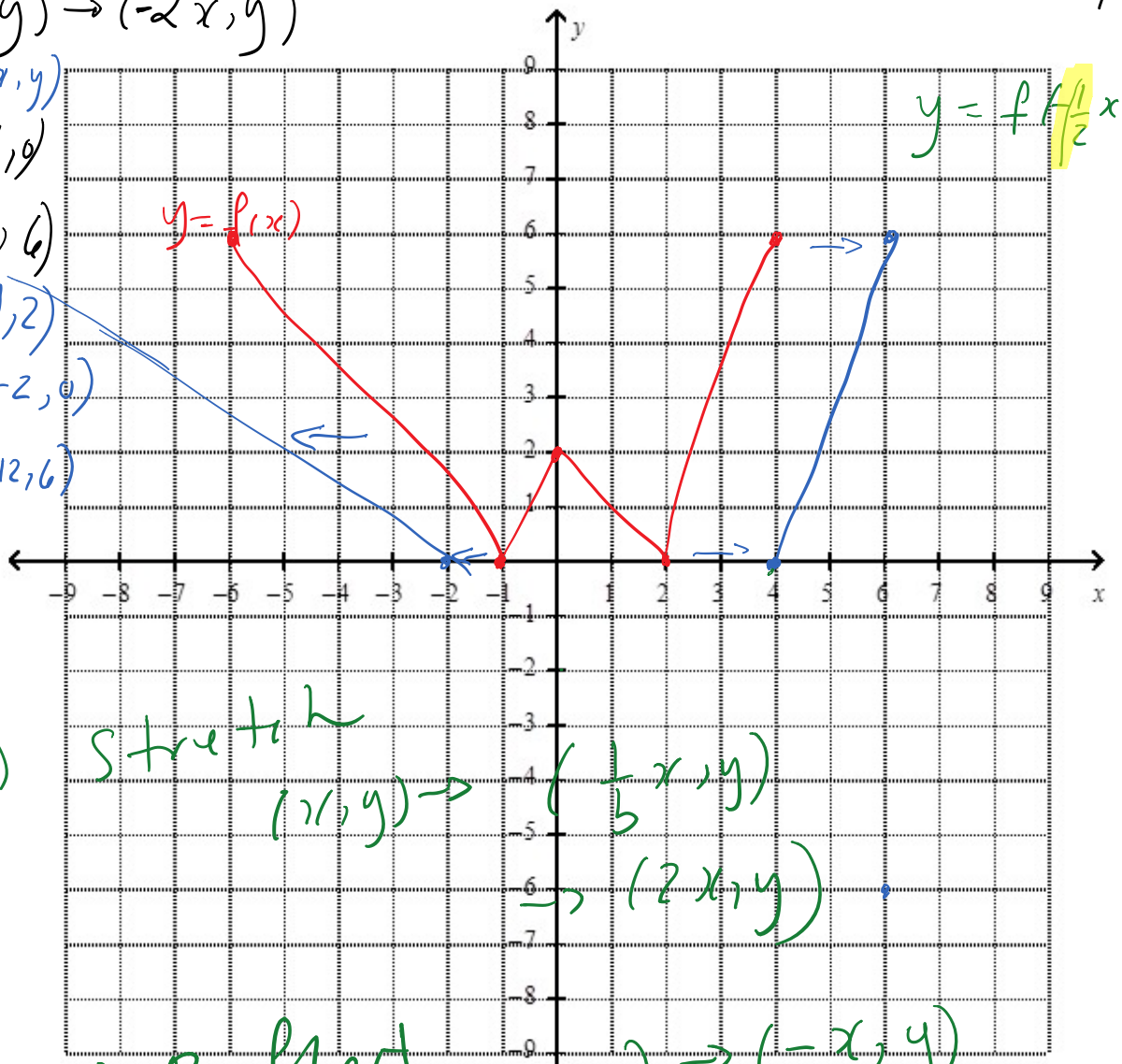
$(2,0) \rightarrow (4,0)$

$(4,6) \rightarrow (8,6)$

$(0,2) \rightarrow (0,2)$

$(-1,0) \rightarrow (-2,0)$

$(-6,6) \rightarrow (-12,6)$



1) Stretch $(x,y) \rightarrow \left(\frac{1}{b}x, y\right)$

$\Rightarrow (2x, y)$

2) Reflect $(x,y) \Rightarrow (-x, y)$