1 block 2

February 24, 2019 7:23 PM

Day 1: Making Conjectures: Inductive Reasoning (1.1)

Big Ideas: In the world around us, we look for patterns. We make *conjectures* about patterns. It is important to see all types of problems from different perspectives. We can use *inductive and deductive reasoning* to make assumptions and to validate an argument.

Pattern: a r e peating order of objects or numbers. We can make predictions about what will come next in a pattern.

Conjecture: an unproven conclusion ("hypotheses") based on specific cases

Inductive reasoning: draw a general conclusion by observing patterns & identifying properties in examples.

To come up with a conjecture, look for patterns in specific examples:

Example 1: Make a conjecture about the product of two odd integers.

$(1)(3) = 3$
$(3)(5) = 15$
$(7)(11) = 77$

↳ x ↳ 1, 3, 5, ...

"The product of 2 odd integers is odd."

Example 2: Make a conjecture about the difference between consecutive perfect squares.

↳ minus

perfect squares

$1^2 = 1$
$2^2 = 4$ } $4-1 = 3$
$3^2 = 9$ } $9-4 = 5$
$4^2 = 16$ } $16-9 = 7$
$5^2 = 25$ } $25-16 = 9$
⋮ ⋮

"The difference between consecutive perfect squares is an odd # and it increases by 2."

Example 3: Ms. Kamber is marking a quiz on inductive reasoning. She spilled chocolate milk on the quiz. A conjecture reads as follows:

"The sum of three consecutive integers is always three times the..." Determine the missing portion using inductive reasoning.

add

?

middle number one!

$1 + 2 + 3 = 6$

$2 + 3 + 4 = 9$

$3 + 4 + 5 = 12$

$4 + 5 + 6 = 15$

Example 4: What conjecture(s) can be made about the figures shown?

Figure 1

Figure 2

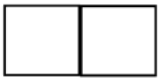


Figure 3

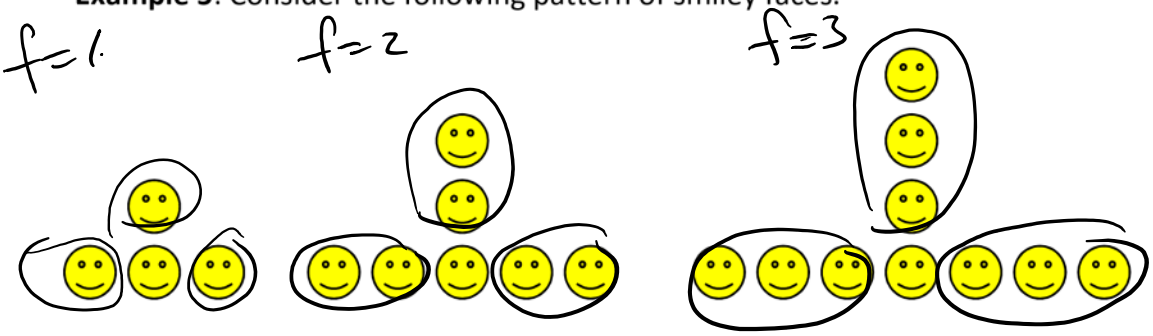
Figure 4

Figure 5

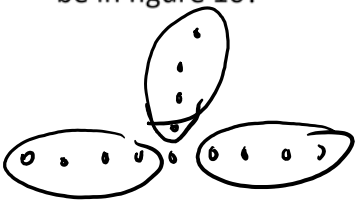
FIGURE NUMBER, *f*	NUMBER OF SIDES, *n*
1	
2	
3	
4	
5	
6	
20	

Example 5: Consider the following pattern of smiley faces.

f=1 f=2 f=3



a) Draw the next (4th) figure in the pattern above. How many smiley faces will be in figure 10?



The 10th has 31.

10

10 10

b) Construct a table of results and make a conjecture relating the figure number and the number of smiley faces.

Figure number, *f*	Number of smiley faces, *n*
1	4
2	7
3	10
4	13
10	31

+3 +3

$3f + \boxed{1} = n$

$3f + \square = n$

$3(1)+1 = 4$ ✓

$3(2)+1 = 7$ ✓

c) Use your conjecture to determine the number of smiley faces in Figure 58.

↳ n? ↳ f=58

$3f + 1 = n$

$3(58) + 1 = \boxed{175.}$

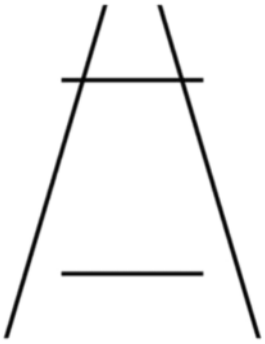
Assignment: Sec. 1.1, p. 12 #3, 8, 9, 11, 12, 13, 16.

Day 2: Exploring the Validity of Conjectures (1.2)

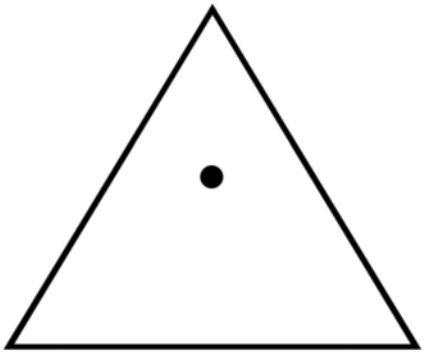
A conjecture is made through observing examples.

Even if you have multiple examples of a conjecture being true, all you need to prove the conjecture is NOT true is one false example (counterexample)!

On your first impression…

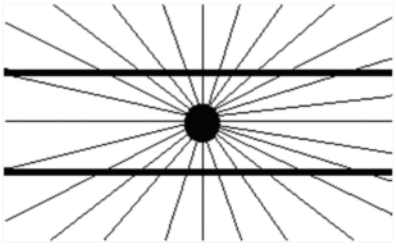


Make a conjecture about the horizontal lines.

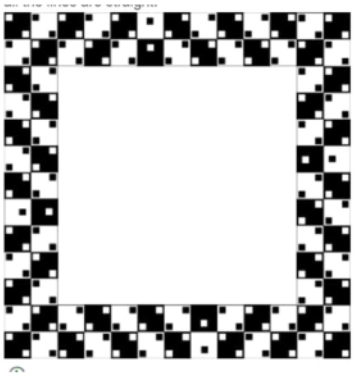


Make a conjecture about the dot (is it halfway down the triangle?)

Look at the Sun. Are these lines Parallel? or Are they Curved?



Make a conjecture about the dark lines.



Make a conjecture about the squares and lines.

Examples:

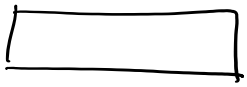
Are the following conjectures valid? If not, find a **counterexample**.

TRUE?

→ show it's FALSE

a) All four-sided figures are squares.

False: counterexample:



b) All birds can fly.

False: ostrich

c) Division decreases the number. For example, 12 ÷ 4 =3 and 3 is less than 12.

$12 \div 4 = 3$ ✓

$8 \div 2 = 4$ ✓

False!

Counterexample $12 \div -4 = 3$

no remainder

d) If a number is divisible (dividing whole numbers that produces another whole number) by 2, then it is divisible by 4.

$\frac{12}{2}$ ✓ $\frac{12}{4}$ ✓ $\frac{30}{2}$ $\frac{30}{4}$ ×

30 is divisible by 2 but NOT by 4 → False

e) If a number is divisible by 9, then it is divisible by 3.

$\frac{18}{9}$ $\frac{18}{3\times3}$

TRUE!

f) If the sum of 2 numbers is negative, then both the numbers are negative.

? + ? = neg?

Counterexample: $1 + -2 = -1$

FALSE

g) If I live in Vancouver (Canada), then I live in British Columbia.

TRUE

h) If I live in British Columbia, then I live in Vancouver (Canada).

False I live in Burnaby.

Day 3: Using Reasoning to Find a Counterexample to a Conjecture (1.3)

Counterexample:

_____

Example 1:

Tom and Jerry were given the following number pattern by their teacher.

$2^2+1=5$ (prime number)

$4^2+1=17$ (prime number)

$6^2+1=37$ (prime number)

$10^2+1=101$ (prime number)

a) Tom made the following conclusion:

"When an even number is squared and one is added, the result is a prime number (a natural number that is greater than 1 and can only be divided by 1 and itself)

Tom's conclusion is a _____ based on _____

b) Jerry investigated Tom's conclusion further. Does Jerry agree or disagree with Tom? Explain.

Example 2:

Bart and Lisa were discussing what conjecture could be made regarding the prime number pattern shown.

3+5=8

5+7=12

7+11=18

13+17=30

19+23=42

29+31=60

Their conjecture: "The sum of two prime numbers is an even number."

Is this a valid conjecture? If not a) provide a *counterexample* and b) *revise* the conjecture so it is true for all prime numbers.

Example 3:

Find a counterexample to this conjecture.

"Natural numbers can be written as the sum of consecutive numbers." Here are some examples:

4+5=9

3+4+5=12

4+5+6+7=22

Example 4:

Here is an interesting numeric pattern:

	A	B
1	1(8)+1	9
2	12(8)+2	98
3	123(8)+3	987
4	1234(8)+4	9876

Find a counterexample that breaks the pattern.

Example 5:
Are the following conjectures valid? If it is not, find a **counterexample**.

a) Every rectangle is a square.

b) Every square is a rectangle.

c) If two angles are acute, their sum is less than 180˚.

d) The square of a number is always greater than the number.

e) If one is added to an odd number, then the result is always an even number.

f) A number that is not positive is negative.

In summary:

1. You need only _____ counterexample to _____ a conjecture!

Assignment: Sec 1.3, p. 22 #1 (pick any 3). Pick any 4 out of: 3-6, 12, 14. Do #10 (how is #10 different from the other questions?)

Day 4: Proving conjectures: Deductive reasoning, Part 1 (1.4)

Mathematical proof: _____

_____

Generalization: _____

_____

Deductive Reasoning: _____

_____

_____

Important "tricks" to use for deductive reasoning proofs!:

A NUMBER:

- Use ___ for a general number. If they are talking about 2 unrelated numbers, use ___ and ____.
- Write "L _ _ *x* be ...." to explain your number.

EVEN NUMBER: Any integer multiplied by 2 is an even number.

- This means that ____ or __(any combination of variables and coefficients) will **always** be _____.

2

ODD NUMBER: If you add 1 to any even integer you will get an odd number.

- This means that _____ or __(any combination of variables and coefficients) + __ will **always** be _____.

3

CONSECUTIVE NUMBERS: These follow each other in numerical order.

- This means that __, _____, _____, and _____ are 4 numbers that come one after the other numerically.
- Consecutive *even* numbers: _____, _____, _____, _____
- Consecutive *odd* numbers: _____, _____, _____, _____

2, 3, 4

Finishing a Proof:

If proving an answer is...	**.. it should look like ....**
Even	__(any combination of variable terms)
Odd	__(any combination of variable terms)_____
Divisible by 3	__(any combination of variable terms)
Divisible by 4	__(any combination of variable terms)
etc.	etc.

Example 1:

Choose a number. Using your number:

1. Multiply by 6
2. Add 14 and divide by 2
3. Add 5 and divide by 3
4. Subtract by your number

Conjecture:

Proof:

Example 2:

Prove that the sum of two consecutive integers is always odd.

Let x = _____

Example 3:
Prove that the difference between consecutive perfect squares is always an odd number.

Example 4: What can you deduce?!
All monkeys are mammals. All mammals are vertebrates. Curious George is a monkey. What can be deducted about Curious George?

Transitive Property:

_____

_____

Example 5: What can you deduce?!
All humans are mortal. Jane is human. What can be deducted about Jane?

Example 6: What can you deduce?!
Jamaica (J), Trinidad-Tobago (T), Barbados (Bar) and Bahamas (Bah) are four countries in the Caribbean. All the following statements about their land areas are true. List the countries in order of increasing size.

- Barbados is smaller than Trinidad-Tobago.
- Bahamas is neither the largest nor the smallest.
- At least two countries are larger than Trinidad-Tobago.

Assignment: Sec. 1.4, p. 31 #2, 4, 5, 7 (number trick), 10 (Optional: 15, 19).

Day 5: Proving conjectures: Deductive reasoning, Part 2 (1.4)

When we make a conclusion based on statements that we accept as _____ we are using d_____ r_____. In proofs using deductive reasoning, we use expressions (e.g., for a number, even number, odd number, consecutive numbers, etc.) and algebra.

Statements that we know are true:

Any integer multiplied by 2 is an even number.
-This means that *2x* or *2*(any combination of variables and coefficients) will **always** be even.
If you add 1 to any even integer you will get an odd number.
-This means that *2x+1* or *2*(any combination of variables and coefficients) *+1* will **always** be odd.
Consecutive numbers follow each other in numerical order
-This means that *x, x +1, x+2, x+3* are 4 numbers that come one after the other numerically.
- consecutive even numbers*:2x, 2x+2, 2x+4, 2x+6*
- consecutive odd numbers *2x+1, 2x+3, 2x+5, 2x+7*

Finishing a Proof:

If proving an answer is…	**.. it should look like ….**
Even	__(any combination of variable terms)
Odd	__(any combination of variable terms)_____
Divisible by 3	__(any combination of variable terms)
Divisible by 4	__(any combination of variable terms)
etc.	etc.

Example 1: Use deductive reasoning to prove that the sum of an odd number and an even number is always odd.

Example 2: Prove that the square of an even integer is always even

Example 3: Prove that the result of the number trick below is always the number you start with.

- Choose a number
- Add 2
- Multiply by 3
- Subtract 6
- Subtract your original number
- Divide by 2

Example 4: The sum of a two-digit number and its reversal is a multiple of 11

Assignment: Deductive Reasoning Worksheet

Day 6: Proofs that are Not Valid (1.5)

Invalid proof: _____

_____

A proof is invalid if it has/uses:

- Divides by zero
- Calculation error
- Reasoning error
- Circular reasoning
- Is based on a false assumption
- Violates a p_____ (true statement)

Example 1: Is this a valid proof? If not, find the error in reasoning:

Thomas is a college student. All college students dislike studying. Therefore, Thomas dislikes studying.

Example 2: Kimberley claims she can prove that 2 = 3. Show that she has written an invalid proof.

Her proof:

$x + y = z$

$3x - 2x + 3y - 2y = 3z - 2z$

$3x + 3y - 3z = 2x + 2y - 2z$

$3(x + y - z) = 2(x + y - z)$

$\frac{3(x + y - z)}{x + y - z} = \frac{2(x + y - z)}{x + y - z}$

$3 = 2$

What is wrong?!

Example 3: Is this a valid proof? If not, find the error in reasoning:

Kurt claims to have proven that 2 = 1. For each statement in his proof, determine if it is valid.

Statement	Reason	Valid?
$x = y$, where $x, y \neq 0$	Given	
$x^2 = xy$	Multiply both sides by *x*	
$x^2 - y^2 = xy - y^2$		
$(x - y)(x + y) = y(x - y)$	*Factor out (x-y)*	
$\frac{(x-y)(x+y)}{x-y} = \frac{y(x-y)}{x-y}$ $x + y = y$	_____ both sides by (x-y)	
$y + y = y$	Substitute *y* for *x* since *x = y*	
$2y = y$	Simplify	
$2 = 1$	_____ both sides by *y*	

Example 4: What error, if any, occurs in the following proof?

2 = 2

4(2) = 4(1 + 1)

4(2) + 3 = 4(1 + 1) + 3

8 + 3 = 6 + 3

11 = 9

Example 5: Mark claims that -3 = 3

Proof: Assume -3 = 3

$(-3)^2 = 3^2$

9 = 9

Therefore: -3 = 3

Where did Mark go wrong?

Example 6: Evan created a number trick in which he always ended with the original number. When he tried to prove the trick, it did not work. Where did the error occur in the proof?!

n	Let n be any number
n + 4	Add 4
2(n + 4)	Multiply by 2
2n + 8	Add 4
2n + 4	Divide by 2
n + 1	Subtract 5

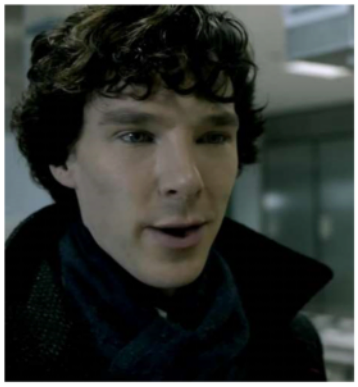
Example 7: Prove this number trick by deduction:

1. Pick a 3-digit number with all three digits being the same (ex., 333, 444, etc.)
2. Add the 3 digits together
3. Divide your three-digit number by the sum you found
4. Your answer is 37.

Assignment: Sec 1.5, p. 42# 1 (like Ex 1 above), 2, 3, 5, 7, 10 (a classic!)

Day 7: Reasoning to solve problems (1.6)

- A **conjecture** is a conclusion based on _____.
- **Inductive reasoning**: Making a c_____ based on e_____, which may be proven by deductive reasoning or disproven by a c_____.
- **Deductive reasoning**: Using known facts or assumptions to draw a logical conclusion.



Example 1: Decide whether the process used is inductive reasoning or deductive reasoning.

a) Show that the sum of 2 even numbers is even by using several examples.

b) No mathematician is boring. You are a mathematician. Therefore, you are not boring.

c) One counterexample proves that a conjecture is false.

d) You show why your statement makes sense.

e) You give evidence that your statement is true.

f) You give six other examples to show that your conjecture is true.

g) What three coins have a value of $0.60?

Using <u>Inductive Reasoning</u> to Solve Problems

- Solve a similar or simpler problem
- Find and use a pattern

Example 2: How many diagonals will a regular (equal sides/angles) decagon (10-sided shape) have?

Using <u>Deductive Reasoning</u> to Solve Problems

- Develop a logical argument using known facts
- Use a logical conclusion that is the solution to the problem

Example 3: Alec, Brandon, Carl, and Dave are on four sports teams.

- Each play on just 1 team.
- They play football, basketball, baseball, & hockey.
- Brandon is a goalie.
- The tallest player plays basketball.
- The shortest plays baseball.
- Carl is taller than Dave but shorter than Al and Bob.

What sports does each play?

Example 4: Alec, Brandon, Carl, and Dave live in the same apartment. They are a manager, teacher, artist, and musician. Alec and Carl watch TV with the teacher. Brandon and Dave go to the hockey game with the manager. Carl jogs with the manager and teacher. Who is the manager?!

	Manager	Teacher	Artist	Musician
Art				
Bill				
Cecil				
Don				

Example 5: Barry, Larry, Jerry, Terry, and Perry are position on a track meet.

- Terry is 20 m behind Barry.
- Barry is 50 m ahead of Larry.
- Larry is 10 m behind Perry.
- Jerry is 30 ahead of Terry.
- Perry is 50 m behind Jerry.

At this point in the race, who is winning? What is the order of all the racers?

Assignment: Sec 1.6, p. 48 # 1, 3, 5a, 6, 8, 9, 15.

BOARDS Day 4/5: Proving conjectures: Deductive reasoning, Part 1 (1.4)

Example 1:
Prove that the sum of two odd numbers will always be even.

Example 2: All dogs are mammals. All mammals are vertebrates. Scooby Do is a dog. What can be deducted about Scooby Do?

Example 3:
Prove the sum of five consecutive integers is 5 times the median of the numbers.
Hint:

- The median is the number in the middle (if the number are in order).
- Let *x* be the median.
- What expressions would you use for the other 4 numbers?!

Day 6: Proofs that are Not Valid (1.5)

Example 1: Alec is trying to prove the following number trick: *Choose any number. Add 3, double it, add 4, divide by 2, and take away the original number.*

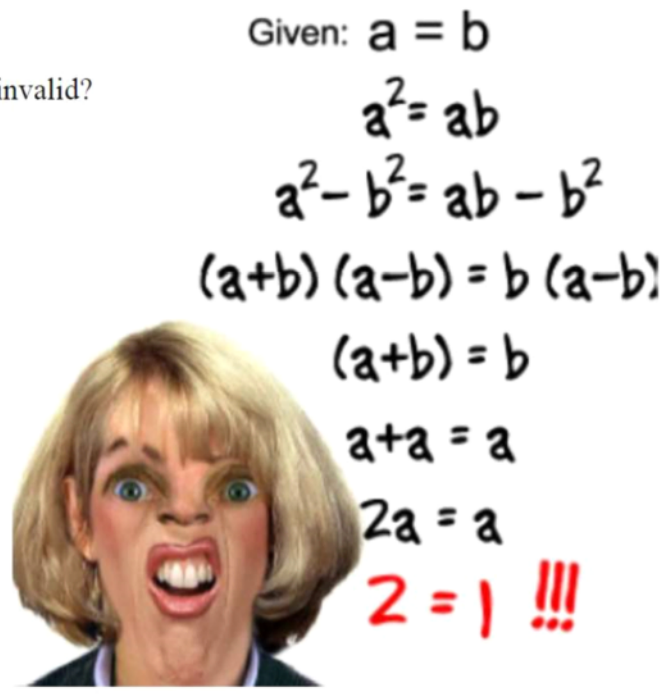
Each time he tries the trick, he ends up with 5. When he tried to prove the trick, it did not work. Where did the error occur in the proof?!

n	Let n be any number
$n + 3$	Add 3
$2n+6$	Double it
$2n + 10$	Divide by 2
$n + 5$	Take away the original number
$n + 1$	Subtract 5

Where was his error?

Example 2:

Why is this proof invalid?



BOARDS Day: **Day 7: Reasoning to solve problems** (1.6)

Ex 1: Rollo, Gorgo, and Zed work in the circus. They are the ringmaster, lion tamer, and clown though not necessarily in that order.

- Zed has red hair
- Rollo has curly hair
- The ringmaster is shorter than Rollo
- The lion tamer is bald.

Who is the clown?

Ex 2: Once upon a time, there was a Wolf, a Rabbit and a basket of Cabbage. You are a farmer and you need to take all three to the Harvest Festival. As you are walking to the Festival, you encounter a river. A boat man will take you across but he only has room for you and one other object (with the Wolf, Rabbit, or Cabbage). How do you get everything across the river safely?

Ex 3: There were 3 pigs, Peony, Petunia, and Poppy. Each lived in a different type of house: straw, wood, and brick. Each gave a different type of treat for Halloween: an apple, popcorn, and chocolate. Use the clues below to figure out which pig lived in each house, and what type of treat each pig handed out.

- Peony did not hand out popcorn
- Poppy does not live in a wood house
- The pig who lives in the straw house handed out popcorn.
- Petunia handed out apples
- The pig who handed out chocolate does not live in a brick house.

Hint: Make a chart to aid in your deduction!

Pig	**House** **Straw Wood Brick**	**Treat** **Apples Popcorn Choc**