1 Intro + Substitution

October 26, 2021 7:40 PM

PRE-CALCULUS 11 Chapters 8-9 – Day 1: SYSTEMS OF EQUATIONS INTRO + SUBSTITUTION

SYSTEMS OF EQUATIONS

$$\begin{cases} 4x + 2y = 9 \\ 3x - 2y = 10 \end{cases}$$

System of equations: a collection of **two** or more equations with the same variables.

Linear-quadratic system of equations: 1 line + 1 parabola

Quadratic-Quadratic system of equations: 2 parabolas

To solve any system of equations, find all values of the variables that *satisfy every equation in the system*.

↳ make the equations true

The solution of a system of equations can be found:

- ***graphically***, or
- algebraically - either by the **substitution** method or the **elimination** method.

SOLVING SYSTEMS OF EQUATIONS BY GRAPHING

To solve a system of equations graphically:

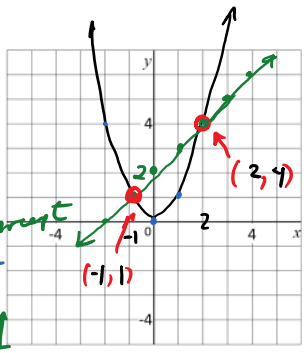
1. *Graph* each equation on the same coordinate plane.
2. Find all the points that are common to all of the graphs, i.e. **look at the point(s) of intersection!**
3. *Verify* your solutions by substitution.

Example 1: Solve this system of equations graphically.

$y = x^2$ and $y = x + 2$

basic parabola
vertex: (0,0)
Graph → 1 ↑ ax1 ax3 ax5.

$y = mx + b$
y-intercept = 2
m = slope = rise/run = 1



what points are in common?
m = 1
{(-1,1), (2,4)}
SOLUTION!

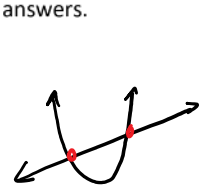
Verify: (-1,1)
$y = x^2$
$= (-1)^2$
$1 = 1$ ✓

$y = x + 2$
$= -1 + 2$
$1 = 1$ ✓

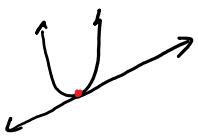
Verify (2,4)
$4 = 2^2$
$4 = 4$ ✓

$4 = 2 + 2$
$4 = 4$ ✓

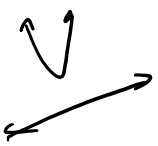
Example 2: How many solutions can a linear-quadratic system have? Illustrate your answers.



2 solutions

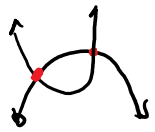


1 solution

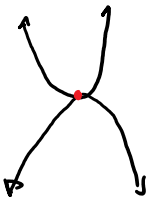


No solution

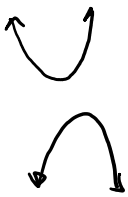
Example 3: How many solutions can a quadratic-quadratic system have? Illustrate your answer.



2 solutions



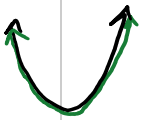
1 solution



No solution

Ex: When solving for x, you get something FALSE

eg. $4 = -7$



2 parabolas on top of each other

infinitely many solutions!

Ex: When solving for x, you get something that's always TRUE

eg. $3 = 3$ ✓

SOLVING SYSTEMS OF EQUATIONS ALGEBRAICALLY WITH THE SUBSTITUTION METHOD

Steps to solve a system of equations algebraically using The Substitution Method:

$$3x + y = 5$$
$$2x + 4y = -10$$

1. Rewrite one of the equations in terms of *y*. (Rewrite the simpler equation.)

$3x + y = 5$
$y = -3x + 5$
$2x + 4y = -10$

2. Take the expression equal to that variable and *substitute* it into the other equation; the result should be a single equation with a single variable.

Replace
=> end up with 1 variable, x!

3. Solve this equation; find the roots - the values of this first variable.

4. *Substitute* each of these roots into an equation with both variables - one at a time; each of these roots will produce an equation with the second variable.

5. Solve these equations; find the value of the second variable.

Example 1: Solve this system of equations BY SUBSTITUTION.

$x^2 - y = 0$ and $x - y = -2$

quadratic linear

Steps:

- Isolate for *y* in the linear function
- Substitute expression for *y* into the quadratic function.
- Solve this quadratic equation.
- Substitute each of these *x*-values Into the linear function to find the Corresponding *y*-values.

$x - y = -2$ Rewrite to get an expression for y.
$+2 \quad +y \quad +2+y$
$x + 2 = y$
$y = x + 2$

$x^2 - y = 0$ given
$x^2 - (x+2) = 0$ Substitute $x+2$ for y
$1x^2 - 1x - 2 = 0$

$a \cdot c = 1 \cdot -2 = -2$
$b = -1$
$(x-2)(x+1) = 0$
$x - 2 = 0$ OR $x + 1 = 0$
$+2 \quad +2$
$x = 2$ | $x = -1$

Could this example be solved with the substitution method using different decisions?

Yes.

Plug in $x = 2$ to either equation to get y:
$y = x + 2$
$= 2 + 2$
$y = 4$ $(2, 4)$

Plug in $x = -1$:
$y = x + 2$
$= -1 + 2$
$y = 1$ => $(-1, 1)$

Solution: $\{(2,4), (-1,1)\}$

Example 2: Solve using The Substitution Method. Find the *exact* values.

Eq① $y = x^2 + 2$ and Eq② $2x - y + 1 = 0$

Substitute Eq① into Eq②

$2x - (x^2 + 2) + 1 = 0$
$2x - x^2 - 2 + 1 = 0$
$-x^2 + 2x - 1 = 0$

Can't factor a negative 1!
Mult. each term by -1!

$x^2 - 2x + 1 = 0$
$(x-1)(x-1) = 0$
$x = 1$

Plug $x = 1$ into either eq. to get y!
$y = x^2 + 2$

Plug $x=1$ into either eq. to get y!

$$y = x^2 + 2$$
$$= (1)^2 + 2$$
$$= 3$$

$\{(1,3)\}$

Assignment: Sec 8.2, p. 451 #1-3, 8, 13, 14

b) $y = (x + 1)^2 - 4$ and $y = -2x^2 + 7$ (challenge) REMOVED BECAUSE very long answer (Ms. Chang's notes)

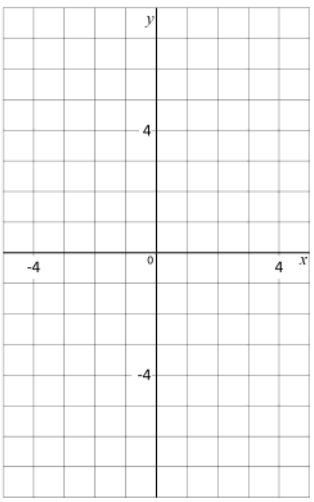
Assignment: Sec 8.1, p. 435, #2, 3ab, 4abd, 5b, 6, 8, 13, opt: 20 (need graph paper for #4)

DO NOT PRINT
HW: p. 435: #2-4abd, 6-8, 13, 20

Exercise: Solve graphically; round to one decimal place if necessary. **Verify your solutions.**

a) $y = x^2 - 4x + 2$

$3x + 3y - 6 = 0$



b) $y = (x + 1)^2 - 4$

$y = -2(x + 1)^2 - 1$