

1 Substitution (8.2)

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PRE-CALCULUS 11 Chapters 8-9 – Day 2: SOLVING SYSTEMS OF EQUATIONS ALGEBRAICALLY (Part 1)

→ solve more than 1 equation

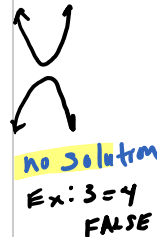
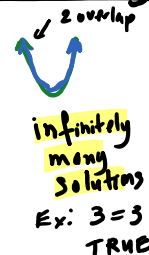
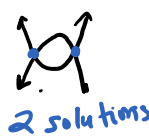
SOLVING SYSTEMS ALGEBRAICALLY

Quadratic - Quadratic System

The solution of a system of equations can be solved:

~~graphically, or~~

- **algebraically** - either:
 - with the **substitution method**, or
 - with the **elimination method**.



SOLVING SYSTEMS OF EQUATIONS WITH THE SUBSTITUTION METHOD

To solve a system of equations algebraically using The Substitution Method:

1. **Solve** ^{→ Rewrite} one of the equations for one of the variables; choose carefully.
2. Take the expression equal to that variable and **substitute** it into the other equation; the result should be a single equation with a single variable.
3. **Solve** this equation; find the roots - the values of this first variable.
4. **Substitute** each of these roots into an equation with both variables - one at a time; each of these roots will produce an equation with the second variable.
5. Solve these equations; find the value of the second variable.

Example 1: Solve this system of equations BY SUBSTITUTION.

$$x^2 - y = 0 \text{ and } x - y = -2$$

Steps: **Isolate**

- Solve for y in the linear function
- **Substitute expression for y** into the quadratic function.
- Solve this quadratic equation.

$$x^2 - y = 0$$

$$x - y = -2$$

$$y = x + 2$$

$$x^2 - (x + 2) = 0$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

- **Substitute** each of these x -values into the linear function to find the corresponding y -values.

Easier to use $y = x + 2$

If $x = 2$, $y = 2 + 2 = 4$

If $x = -1$, $y = -1 + 2 = 1$

$$\text{Soln } \{ (2, 4), (-1, 1) \}$$

Could this example be solved with the substitution method using different decisions?

Yes!

Example 2: Solve using The Substitution Method. Find the *exact* values.

$$\textcircled{1} y = x^2 + 2 \quad \text{and} \quad \textcircled{2} 2x - y + 1 = 0$$

Rewrite $\textcircled{2} \Rightarrow \textcircled{3} y = \boxed{2x+1}$

Substitute $\textcircled{3}$ into $\textcircled{1}$.

$$\boxed{y} = x^2 + 2$$

$$2x+1 = x^2 + 2$$

$$\begin{array}{r} -2x \quad -1 \\ x^2 - 2x + 1 = 0 \end{array}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(x-1)^2 = 0$$

$$(x-1)(x-1) = 0$$

$$\boxed{x=1}$$

Find corr. y :

$$\begin{aligned} y &= 2x+1 \\ &= 2(1)+1 = \boxed{3} \end{aligned}$$

$$\text{Soltn: } \{ (1, 3) \}$$

Assignment: Sec 8.2, p. 451 #1-3, 8, 13, 14

Solve by SUBSTITUTION:

a) $y = x^2 - 4x + 2$ and
 $3x + 2y - 11 = 0$

b) $y = x^2 - 4x + 2$ and
 $x - 2y - 8 = 0$

(Just in case: quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

Ans: a) $\{(-1, 7), (\frac{7}{2}, \frac{1}{4})\}$ b) no solution

b) $y = (x + 1)^2 - 4$ and $y = -2x^2 + 7$ (challenge) REMOVED BECAUSE very long answer (Ms. Chang's notes)