1 Working with Radicals: Add Subtract

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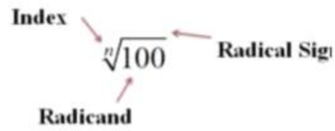
Learning Outcomes: I can:

- identify the **index** and **radicand** of a **radical**
- write **entire radicals as mixed radicals**, e.g., $\sqrt{48} = 4\sqrt{3}$.
- write **mixed radicals** as **entire radicals**, e.g., $4\sqrt{3} = \sqrt{48}$.
- **simplify radical expressions** e.g., $\sqrt[3]{40} = 2\sqrt[3]{5}$
- **add** and **subtract** radical expressions.

$\sqrt[n]{\ }$ indicates an nth root.



RADICALS

Radicals are expressions in the form $\sqrt[n]{a}$, which is the "nth root of a".

$\sqrt{\ }$ is the **radical symbol**; n is the **index** (a positive integer); and a is the **radicand**.

$\sqrt[n]{a}$ = a number such that (the number)^n = a

$\sqrt[2]{25} = 5$ because $5^2 = 25$

$\sqrt[3]{8} = 2$ because $2^3 = 8$

$\sqrt[4]{81} = \sqrt[4]{3\cdot 3\cdot 3\cdot 3} = 3$ because $3^4 = 81$

or "quadruplets" escape!

8 → 2, 4 → 2, 2

81 → 9, 9 → 3 3 3 3

Cube Root Jail

- $\sqrt[3]{8}$
  $= \sqrt[3]{2.2.2}$
  $= \boxed{2}$ ("$\sqrt{\ }$")
- If jail is empty, it disappears

Restrictions on Radicands: $\sqrt[n]{a}$

$\sqrt{-25}$ = a number only if (the number)^2 = –25

There is no real number that will satisfy this condition, ∴ $\sqrt{-25}$ is not a real number.

↳ an imaginary number!

- If the index is even, the radicand must be greater than or equal to zero and the radical is positive.

  ≥ 0

  $\sqrt[2]{49} = \sqrt{7\cdot 7} = 7$ index is 2

  $\sqrt[4]{16} = \sqrt[4]{2\cdot 2\cdot 2\cdot 2} = 2$ index is 4

  $\sqrt[\text{even}]{\text{positive or zero}}$

- If **index is** odd, the radicand could be any real number:

  If the radicand is positive, then the radical is positive $\sqrt[3]{27} = \sqrt[3]{3\cdot 9} = \sqrt[3]{3\cdot 3\cdot 3} = 3$

  If the radicand is negative, then the radical is negative $\sqrt[3]{-8} = \sqrt[3]{(-2)(-2)(-2)} = -2$

$\sqrt[\text{odd}]{\text{positive or negative}}$

positive or 0
≥ 0

Example 1: For what values of x is $\sqrt{7-3x}$ a real value?

$7 - 3x \geq 0$ because index is 2 → even (square root)

$-7 \qquad -7$

$-3x \geq -7$

$\frac{-3x}{-3} \quad \frac{-7}{-3}$

$x \leq \frac{7}{3}$

★ If divide by a negative, switch $\geq$ to $\leq$ or $\leq$ to $\geq$!!

SIMPLIFYING SQUARE ROOTS

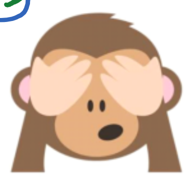
There are rules that can be used to work with radicals.

$\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$ $\qquad a^m \cdot a^n = a^{m+n}$

$\sqrt{5} \times \sqrt{3} = \sqrt{5 \cdot 3} = \sqrt{15}$ $\qquad \frac{\sqrt{45}}{\sqrt{5}} = \sqrt{\frac{45}{5}} = \sqrt{9} = 3$ $\qquad \sqrt[3]{x^2} = x^{2/3}$ $\qquad \sqrt[2]{x^1} = x^{\frac{1}{2}}$

Simplifying ~~square~~ roots:

- the radicand cannot have a factor that is perfect square larger than 1 $\qquad \sqrt{3 \cdot 3} = 3$
- The radicand cannot have a fraction or decimal $\qquad \cancel{\sqrt{2.3}} \qquad \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\cancel{\sqrt{3}}}$
- The denominator cannot be a radical $\qquad \frac{5}{\cancel{\sqrt{2}}}$



Example 2: Simplify $\sqrt{72} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$

72 → 8, 9; 8 → 2, 4; 4 → 2, 2; 9 → 3, 3

entire radical $= 2 \cdot 3\sqrt{2}$

$= 6\sqrt{2}$ mixed radical

An **entire radical** is an expression that is only a radical; $\sqrt{72}$.

A **mixed radical** is an expression that is a product of a non-radical and a radical; $6\sqrt{2}$.

Write $7\sqrt{2}$ as an entire radical: $= \sqrt{2 \cdot 7 \cdot 7} = \sqrt{98}$ entire radical

mixed radical ★

Example 3: For $a \geq 0$ and $b \geq 0$, simplify $\sqrt{48a^6b^9}$

48 → 6, 8; 6 → 2, 3; 8 → 2, 4; 4 → 2, 2

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}$

$= 2 \cdot 2\, a^3 b^4 \sqrt{3b} = \boxed{4a^3b^4\sqrt{3b}}$

Example 4: Simplify $\sqrt[3]{40}$ cube root → triplets escape!

40 → 4, 10; 4 → 2, 2; 10 → 2, 5

$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 5}$

$= 2\sqrt[3]{5}$

Example 5: Simplify $\sqrt[3]{54}$ $= \sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3} = 3\sqrt[3]{2}$

54
6 9
2 3 3 3

Example 6: Simplify $\sqrt[4]{32x^4y^9}$ $= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \, x^4 \cdot y^4 \cdot y^4 \cdot y^1}$

y^9

$= \boxed{2xy^2\sqrt[4]{2y}}$

32
4 8
2 2 2 4
2 2

ADDING AND SUBTRACTING RADICALS

Adding/subtracting monomial terms: only like terms can be combined.

$7x + 5y + 9x$ simplifies to $16x + 5y$

Adding/subtracting radical terms: only terms with the same radical can be combined.

$7\sqrt{2} + 5\sqrt{3} + 9\sqrt{2}$ simplifies to $16\sqrt{2} + 5\sqrt{3}$

Example 7: Simplify $1\sqrt{11} - 8\sqrt{13} - 5\sqrt{11} - 8\sqrt{13}$

$= \sqrt{11} - 5\sqrt{11} - 8\sqrt{13} - 8\sqrt{13}$

$= -4\sqrt{11} - 16\sqrt{13}$

Example 8: Simplify $\sqrt{5} + \sqrt{20} - \sqrt{45}$

20
4 5
2 2

1
3)3
3)9
5)45

$= \sqrt{5} + \sqrt{2 \cdot 2 \cdot 5} - \sqrt{3 \cdot 3 \cdot 5}$

$= 1\sqrt{5} + 2\sqrt{5} - 3\sqrt{5}$

$= 0$

Example 9: Simplify $8\sqrt{6} - 5\sqrt{12} + 2\sqrt{27}$

12
2 6
2 3

27
3 9
3 3

$= 8\sqrt{6} - 5\sqrt{2 \cdot 2 \cdot 3} + 2\sqrt{3 \cdot 3 \cdot 3}$

$= 8\sqrt{6} - 5(2)\sqrt{3} + 2 \cdot 3\sqrt{3}$

$= 8\sqrt{6} - 10\sqrt{3} + 6\sqrt{3}$

$= \boxed{8\sqrt{6} - 4\sqrt{3}}$

Assignment: ~~Sec 5.1, p. 278 #2-4, 5ab, [8-10]ac, 12, 14 opt: 13~~

Simplify:

1) $\sqrt{7}+\ \sqrt{3}$ Can this by simplified?

2) $3\sqrt{2}+\ \sqrt{8}$

3) $\sqrt{18}-\ \sqrt{2}$

4) $4\sqrt{18}-\ \sqrt{8}+3\sqrt{2}-\sqrt{32}$

5) $3\sqrt[3]{81}+\ 3\sqrt[3]{24}$