2 Absolute Value Functions

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PRE-CALCULUS 11 Chapter 7 – Day 2: ABSOLUTE VALUE FUNCTIONS

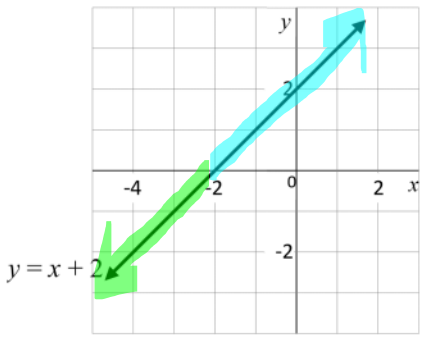
ABSOLUTE VALUE FUNCTIONS

- An ***absolute value function*** has the form $y = |f(x)|$, where *f(x)* is a function.
- The **basic** absolute value function is $y = |x|$.
- The x-intercept of the graph of $y = f(x)$ is a **critical point**. The graph of $y = |f(x)|$ changes direction at this point.
- Points that don't change during a transformation are **invariant points**.
- We will graph the absolute value of linear functions and the absolute value of quadratic functions by transforming their "normal" version.

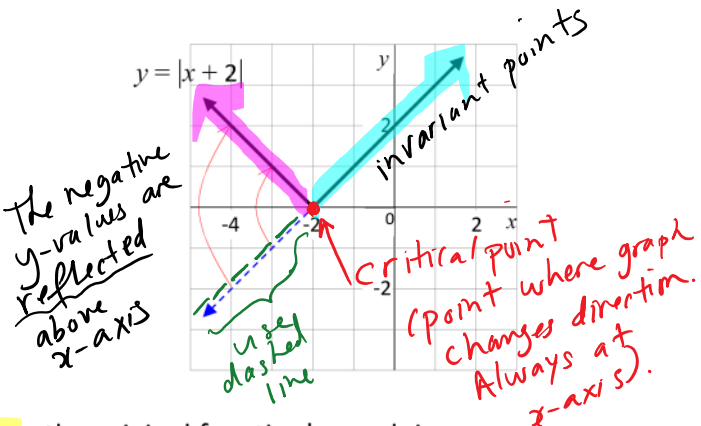
GRAPHING $y = |mx + b|$, THE ABSOLUTE VALUE OF A LINEAR FUNCTION

Example 1: Draw the graph of $y = |x + 2|$ by transforming the graph of $y = x + 2$.

x	$y = x + 2$
–5	–3
–4	–2
–3	–1
–2	0
–1	1
0	2
1	3



x	$y=\|x+2\|$
–5	3
–4	2
–3	1
–2	0
–1	1
0	2
1	3



When absolute value is applied to a function, the original function's graph is transformed - all points below the x-axis (points with negative y-coordinates) are reflected about the x-axis to above the x-axis.

Points that remain unchanged during a transformation are called ***invariant points***.

From last class, the Absolute value $y = |x|$ can be written as: $y = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } < 0 \end{cases}$ ← x is 0 or pos. ← x is neg. (opposite)

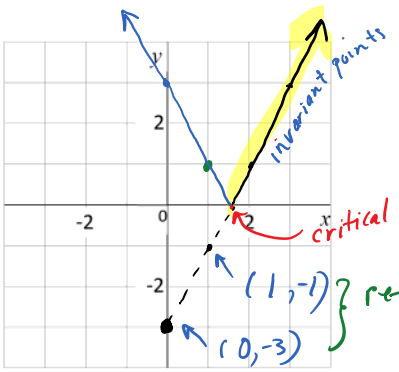
If we apply the mathematic definition of absolute value, $y = |x+2|$ can be written as:

$y = \begin{cases} x+2 & \text{if } x+2 \geq 0 \\ -(x+2) & \text{if } x+2 < 0 \end{cases}$ which simplifies to $y = \begin{cases} x+2 & \text{if } x \geq -2 \\ -x-2 & \text{if } x < -2 \end{cases}$ critical point

A function written in this format is said to be in ***piecewise notation***.

Remember to consider characteristics of graphs we have seen before: the domain and range, the vertex, the axis of symmetry.

Example 2: Draw the graph of $y = |2x - 3|$



① Draw the graph of $y = 2x - 3$.

$y = mx + b$; $y\text{-int} = -3$; $\text{slope} = m = 2 = \frac{2}{1} = \frac{12}{\to 1}$

② Use SOLID LINE for points on & above x-axis ⇒ invariant points

③ Use DASHED LINE below x-axis (because they're not part of abs. value function).

④ Reflect the points that are below the x-axis (SOLID LINE).

$y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

so $y = |2x - 3|$ becomes $y = \begin{cases} 2x-3 & \text{if } x \geq \frac{3}{2} \\ -(2x-3) & \text{if } x < \frac{3}{2} \end{cases}$ (critical point)

To find critical point:

$2x - 3 = 0$

$+3 \quad +3$

$\frac{2x}{2} = \frac{3}{2}$

$x = \frac{3}{2}$

$y = \begin{cases} 2x-3 & \text{if } x \geq \frac{3}{2} \\ -2x+3 & \text{if } x < \frac{3}{2} \end{cases}$

GRAPHING $y = |a(x-p)^2 + q|$, THE ABSOLUTE VALUE OF A QUADRATIC FUNCTION

Example 3: Draw the graph of $y = |x^2 - 4x + 3|$ by transforming the graph of the

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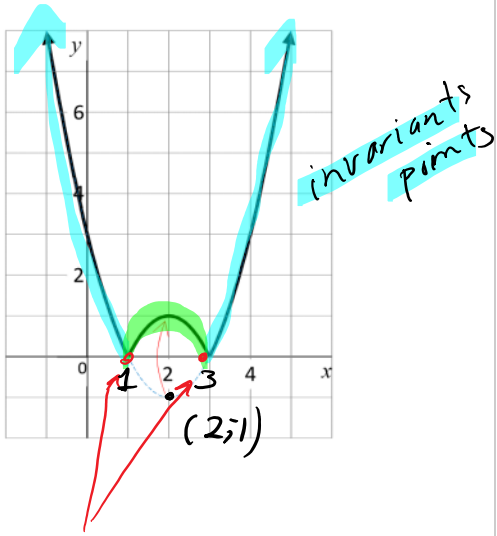
Example 3: Draw the graph of $y = |x^2 - 4x + 3|$ by transforming the graph of the quadratic function and then verify your graph by writing the absolute value function as a piecewise function.



○ Draw the graph of $y = x^2 - 4x + 3$

- Convert to vertex form (by completing the square)

$\frac{b}{2} = \frac{-4}{2} = -2 \rightarrow (-2)^2 = 4$

$y = x^2 - 4x + 4 - 4 + 3$

$= (x-2)^2 - 1$ vertex is $(2,-1)$

- Factor to find x-intercepts: (roots)

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\therefore x = 1$ or $x = 3$

↳ x-intercepts $(1,0)$ & $(3,0)$ are the CRITICAL POINTS (where graph changes direction)

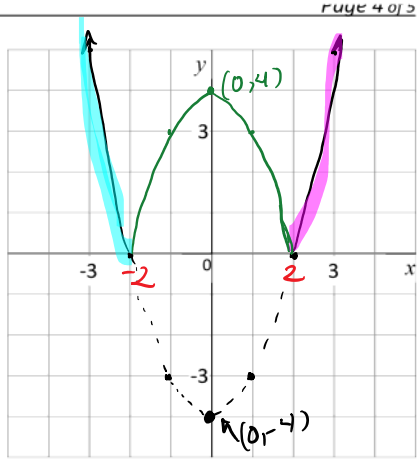
- Use SOLID LINE above x-axis
  Use DASHED LINE below x-axis
- Reflect the points below the x-axis.

using roots (critical points)

so $y = |x^2 - 4x + 3|$ becomes $y = \begin{cases} (x-2)^2 - 1 & \text{if } x \leq 1 \text{ or } x \geq 3 \\ -(x-2)^2 + 1 & \text{if } 1 \leq x \leq 3 \end{cases}$

$-(x-2)^2 - 1$

Example 4: Draw the graph of $y = |x^2 - 4|$ by transforming the graph of the quadratic function.
Verify your graph by writing the absolute value function as a piecewise function.



$y = x^2$ Basic quadratic

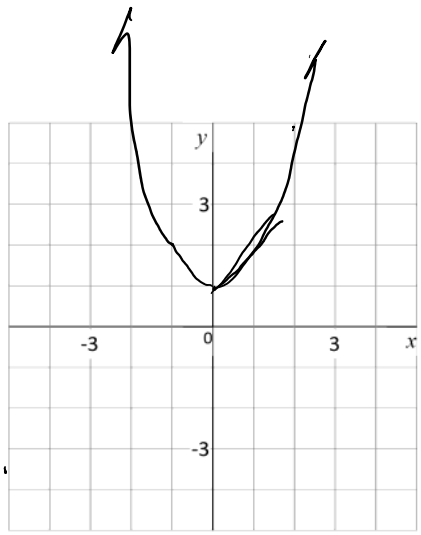
$y = x^2 - 4$ is Basic quadratic translated DOWN 4 units

ROOTS

Piecewise Notation

$$y = \begin{cases} x^2 - 4 & \text{if } x \leq -2 \text{ or } x \geq 2 \\ -(x^2 - 4) & \text{if } -2 < x < 2 \end{cases}$$

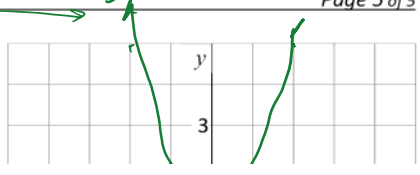
Example 5: Draw the graph of $y = |x^2 + 1|$ by transforming the graph of the quadratic function.



No change! because $y = x^2 + 1$ is all above x-axis!

$y = |-x^2 - 1|$

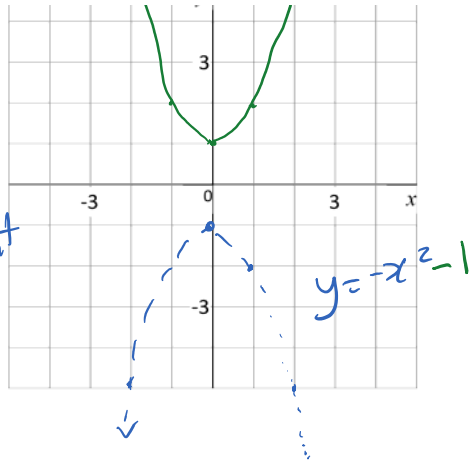
Example 6: Draw the graph of $y = |-x^2 - 1|$ by transforming the graph of the quadratic function.



Example 6: Draw the graph of $y = |-x^2 - 1|$ by transforming the graph of the quadratic function.

$y = -x^2 - 1$

↑ shift basic parabola down 1 unit



Assignment: Sec 7.2, p. 375 #1-5, 6ac, 7ab, 8ae, 9ac, 10a, 11ab, 12, 13, 22