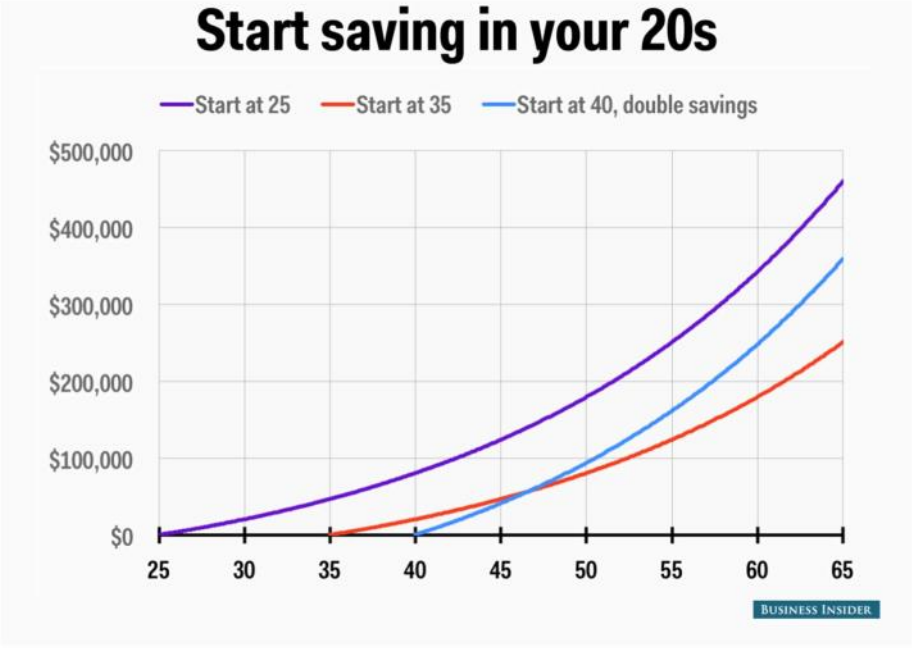
2 Compound Interest

June 6, 2023 11:40 AM

Math 11/12: Financial Literacy — Day 2: Compound Interest: Future Value



COMPOUND INTEREST

With simple interest, the principal at the beginning of the second year is the same as the principal at the beginning of the first year. In compound interest, the interest earned during the first year is added to the original principal to form a new LARGER principal.

Simple vs. Compound interest?! Say a bank offers you two types of savings bonds:

- Simple Interest bond paying 6% per year. $r = 0.06$
- Compound Interest bond paying 6% per year compounded annually

Complete the following table to see which is better:

$I = Prt$
$= (2000)(0.06)(1)$
$= \$120$

YEAR	Principal ($)	**Simple** Interest ($)	Ending Simple Interest Value ($)	*Principal ($)*	*Compound Interest ($)*	*Ending Compound Interest Value ($)*
1	2,000	120	2120	2,000	120	2120
2	2000	120	2240	2120	127.20	2247
3	↓	↓	2,360	2247	134.82	2382.03
4			2,480	2382.03	142.92	2,524.95

$I = Prt$
$= (2120)(0.06)(1)$
$= \$127.20$

Which type of bond grows more quickly? compound interest!

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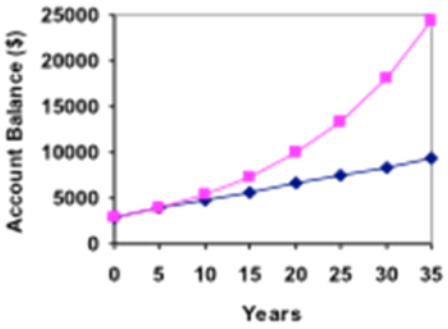
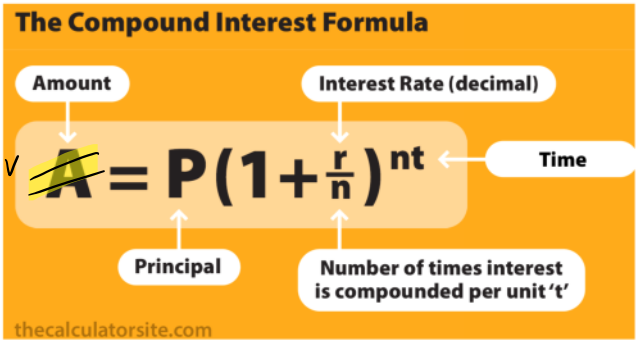


Compound interest (exponential!)

Simple interest (linear)

- Compound Interest: The interest that is earned on both the principal and the accumulated interest
- **Future Value** (FV) or Accumulated Amount (A): How much an investment is worth after a certain amount of time

FORMULAE:



FV

$I = FV - P$

n = # of compounding periods per year

n, Compounding Periods per year

n is the number of paid periods in a year,
(i.e., how often they will pay YOU the interest!)

How often is the interest calculated ("compounded") per year?	*n*
annually	1
semi-annually	2
quarterly	4
monthly	12
weekly	52
bi-weekly	26
daily	365

56 ÷ 2 = 28

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Example 1: Determine the future value if you were to have invested $7,800 at 6.75% compounded annually for 15 years.

$P = 7,800$

$r = 6.75\% = 0.0675$

$t = 15$ years

$n = 1$

$$FV = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 7800\left(1 + \frac{0.0675}{1}\right)^{(1)(15)}$$

$$= 7800(1.0675)^{15}$$

$$FV = \$20,778.44$$

exponent

x^n y^n x^y $\wedge$

exponents 1st!

How much interest was earned (the compounded interest)?

$$I = FV - P = \$20,778.44 - 7,800 = \$12,974.44$$

Example 2: Ishaan invested $23,000 inheritance in an account that earns 13.6%, compounded semi-annually. The interest rate is fixed for 10 years. Matt plans to use the money for a down payment on a condo.

a) What is the future value of the investment after 10 years?

$P = 23,000$

$r = 13.6\% = 0.136$

$n = 2$

$t = 10$

$$FV = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 23000\left(1 + \frac{0.136}{2}\right)^{2(10)}$$

$$= 23000(1.068)^{20}$$

$$FV = \$85,733.96$$

b) If the investment had earned simple interest, would the relationship between the principal and the future value have been the same? Explain.

$$I = prt$$

$$= (23000)(0.136)(10)$$

$$I = \$31,280$$

$$FV = P + I = 23000 + 31,280$$

$$= \$54,280$$

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Rule of 72: A simple for estimating the doubling time of an investment.

Number of years to **Double** = 72 ÷ Interest Rate as %

not a decimal!

This rule works best when interest is compounded annually!

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Example 3: Both Bob and Chris invested $5,000 in separate accounts. Bob's earns 8%, compounded annually, while Chris' earns 9%, compounded annually.

a) Estimate the doubling time for each.

Bob:

years to double = $\frac{72}{8}$

= 9 years

Chris:

years to double = $\frac{72}{9}$

= 8 years

b) Does the Rule of 72 really work?!? Estimate the future value of an investment of $5,000 that earns 8%, compounded annually, for 9, 18, and 27 years. How close are your estimates to the actual future values?

BOARDS

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