2 Factoring

June 5, 2021 7:49 PM

FOM 11 Factoring - Day 2: Factoring Polynomials

FACTORING POLYNOMIALS

"To factor" something means to write it as an equivalent product. For example, the factored form of 15 is (3)(5).

$(3)(5) = 15$ ← product; 1 & factors

PLAN
(P+L)(A+N)
PA+PN+LA+LN

Your plan has been foiled

Greatest Common Factor:

Greatest Common Factor (GCF): $ab + ac = a(b + c)$

Definition: the largest factor shared by 2 or more terms

Tip: If the leading term is *negative*, factor out the negative (e.g., **-1**)!

Example 1:

a) What is the GCF of $18x^2$ and $6x$? GCF = $6x$

1 x 18, x, x 1 x 6, x
2 x 9 2 x 3
3 x 6

b) Factor $18x^2 + 6x$: $= 6x(3x + 1)$

Same sign: (+)(+) = (+), (−)(−) = (+)
diff sign: (+)(−) = (−), (−)(+) = (−)

Example 2:

a) Factor $9x + 45$: GCF = 9

$9(x + 5)$

c) Factor $-20x^2 + 44$: (factor out the negative!)

$-4(5x^2 - 11)$

b) Factor $14x^2 - 35x$ GCF = $7x$

$7x(2x - 5)$

Review Simplifying Fractions

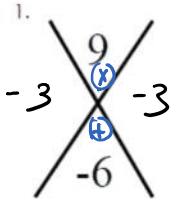
Example 3: Simplify the following:

a) $\frac{2}{8} \div \frac{2}{2} = \frac{1}{4}$ b) $\frac{6}{-8} = \frac{3}{-4} = \frac{-3}{4}$ c) $\frac{6}{-9} = \frac{-2}{3}$

Sum & Product Puzzle: Set 1

In each diagram below, write the two numbers on the sides of the "X" that are *multiplied* together to get the top number of the "X," but *added* together to get the bottom number of the "X."

1.



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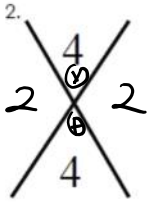

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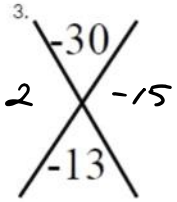

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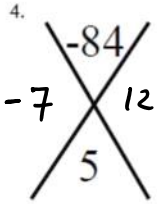

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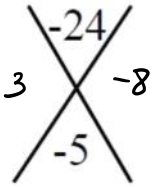

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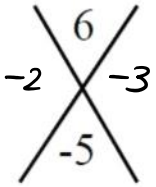

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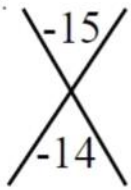
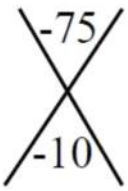


8.



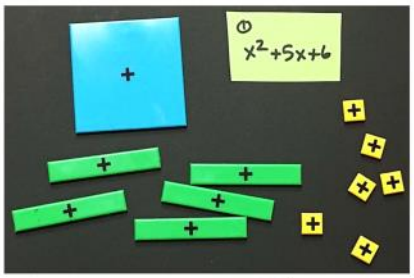
9.

Factoring Polynomials of the form $ax^2 + bx + c$ with Algebra Tiles

Steps:

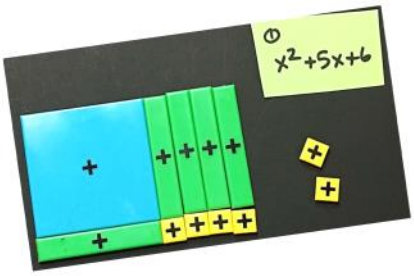
1. Lay out the polynomial with algebra tiles.
2. Arrange the tiles into a rectangle! Your rectangle will have:
   - the x^2 tile(s) in its upper left corner
   - the unit ("1") tiles in the lower right corner
3. Read the dimensions of your rectangle. These are the factors!

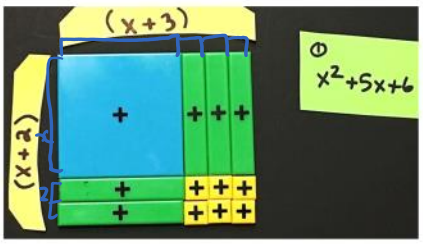
Example: Factor $x^2 + 5x + 6$ (from Scaffoldedmath.com)



Arrange into a rectangle!

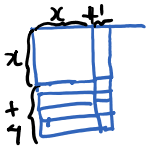
$x^2+5x+6 = (x+3)(x+2)$



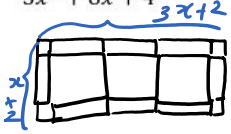


Example 4: Use algebra tiles to factor. Draw the tiles to show your answer.

a) $x^2 + 5x + 4$ $= (x+1)(x+4)$



b) $3x^2 + 8x + 4$ $= (3x+2)(x+2)$



Factoring

➢ Factor out GCF 1st!!
(Factor out -1 if a is neg.!)

$$ax^2 + bx + c$$

$$\square \times \square = ac$$

$$\square + \square = b$$

$$\frac{ax}{\square} \quad \frac{ax}{\square}$$

Reduce columns (like fractions!)

$$(x \quad)(x \quad)$$

Factoring Trinomials of the form $ax^2 + bx + c$ using "Crosshairs" Method

Example 5: $4x^2 - 10x - 24$

Factoring

$ax^2 + bx + c$

□ ✖ □ = ac =

□ ✚ □ = b =

ax	ax
□	□

Reduce columns (like fractions!)

(x)(x)

Step 1: **Factor first**: Is there a Greatest Common Factor (GCF) you can factor out from the 3 terms (other than 1)? If *a* is *negative*, factor out the negative!

GCF = 2 $\quad 2(2x^2 - 5x - 12)$

$a x^2 + b x + c$

Step 2: Identify ***a***, ***b***, and ***c*** in (new) $ax^2 + bx + c$:
Multiply *a* and *c*.

$a = 2 \quad b = -5 \quad c = -12 \quad a \times c = 2(-12) = -24$

Step 3: Find 2 numbers that **multiply** to *ac* **and** add to *b*. (Note: You may need to list the factors of *ac* and remember the **sign law** for multiplication.)

-24, -5

3 x −8 = *ac* = −24

3 + −8 = *b* = −5

Step 4: Draw a "**crosshair**." In the top 2 boxes, write "*ax*". Note: Don't write the "square" (i.e., forget the exponent 2). In the bottom 2 boxes, write the numbers you found in Step 3.

$2x$	$2x$
3	−8

Step 5: Reduce each column. (This is sort of like what you do when you reduce fractions.) In this example, the 2 and 3 in the 1st column will not reduce but the 2 and -8 in the second column can reduce to 1 and -4, as shown.

$2x$	~~$2x$~~ ÷2	$1x$
3	~~−8~~ ÷2	−4

Step 6: You've now got the answer! The 1st column is a factor and the 2nd column is the other factor!

$(2x+3)(x-4)$

Step 7. Check! Apply the distributive property (FOIL) to check your answer. ✓

$2(2x^2 - 8x + 3x - 12)$

$2(2x^2 - 5x - 12)$

GCF

Ex. 6: Factor: $-2x^2 + 2x + 12$

Step 1: Factor first: Is there a Greatest Common Factor (GCF) you can factor out from the 3 terms? If *a* is *negative*, factor out the negative!

GCF = -2 $-2(x^2 - x - 6)$

Step 2: Identify *a*, *b*, and *c* in (new) $ax^2 + bx + c$. Multiply *a* and *c*.

a = 1 *b* = -1 *c* = -6 $a \times c =$ -6

Step 3: Find 2 numbers that **multiply to *ac* and add to *b***.

-6, -1

-3 x 2 = *ac* = -6

-3 + 2 = *b* = -1

Step 4: Draw a "**crosshair**." In the top 2 boxes, write "*ax*". Note: Don't write the "square" (i.e., forget the exponent 2). In the bottom 2 boxes, write the numbers you found in Step 3.

shortcut if a = 1

x	x
-3	2

Step 5: Reduce each column (sort of like what you do when you reduce fractions.)

It's already reduced!

Step 6: You've now got the answer! The 1st column is a factor and the 2nd column is the other factor!

$-2(x-3)(x+2)$

Step 7. Check! Apply the distributive property (FOIL) to check your answer.

Ex. 7: Factor: $10x^2 - 43x - 9$

Step 1: Factor first: Is there a Greatest Common Factor (GCF) you can factor out from the 3 terms? If *a* is *negative*, factor out the negative!

none! keep going!

Step 2: Identify *a*, *b*, and *c* in (new) $ax^2 + bx + c$. Multiply *a* and *c*.

a = 10 *b* = -43 *c* = -9 $a \times c =$ -90

Step 3: Find 2 numbers that **multiply to *ac* and add to *b***.

2 x -45 = *ac* = -90

2 + -45 = *b* = -43

Step 4: Draw a "**crosshair**." In the top 2 boxes, write "*ax*". Note: Don't write the "square" (i.e., forget the exponent 2). In the bottom 2 boxes, write the numbers you found in Step 3.

5 10x	10x 2
1 2	-45 -9

Step 5: Reduce each column (sort of like what you do when you reduce fractions.)

Step 6: You've now got the answer! The 1st column is a factor and the 2nd column is the other factor! $(5x+1)(2x-9)$

Step 7. Check! Apply the distributive property (FOIL) to check your answer.

✓

Ex. 8: Factor: $x^2 - 8x + 12$

Step 1: Factor first: Is there a Greatest Common Factor (GCF) you can factor out from the 3 terms? If *a* is *negative*, factor out the negative!

nom!

Step 2: Identify *a*, *b*, and *c* in (new) $ax^2 + bx + c$. Multiply *a* and *c*.

a = 1 *b* = -8 *c* = 12 *a* × *c* = 12

Step 3: Find 2 numbers that **multiply to *ac* and add to *b***.

-6 x -2 = *ac* = 12

-6 + -2 = *b* = -8

Shortcut because a = 1!!

Step 4: Draw a "**crosshair**." In the top 2 boxes, write "*ax*". Note: Don't write the "square" (i.e., forget the exponent 2). In the bottom 2 boxes, write the numbers you found in Step 3.

Step 5: Reduce each column (sort of like what you do when you reduce fractions.)

Step 6: You've now got the answer! The 1st column is a factor and the 2nd column is the other factor! *Do you see a shortcut?!?*

$(x-6)(x-2)$

Step 7. Check your answer!

Ex. 9: Factor: $2x^2 - 6x - 56$

Step 1: Factor first: Is there a Greatest Common Factor (GCF) you can factor out from the 3 terms? If *a* is *negative*, factor out the negative!

$2(x^2 - 3x - 28)$

Step 2: Identify *a*, *b*, and *c* in (new) $ax^2 + bx + c$. Multiply *a* and *c*.

a = 1 *b* = -3 *c* = -28 *a* × *c* = -28

Step 3: Find 2 numbers that **multiply to *ac* and add to *b***.

4 x -7 = *ac* = -28

4 + -7 = *b* = -3

Step 4: Draw a "**crosshair**." In the top 2 boxes, write "*ax*". Note: Don't write the "square" (i.e., forget the exponent 2). In the bottom 2 boxes, write the numbers you found in Step 3.

x	x
4	-7

Step 5: Reduce each column (sort of like what you do when you reduce fractions.)

Step 6: You've now got the answer! The 1st column is a factor and the 2nd column is the other factor!

$2(x+4)(x-7)$

Step 7. Check! Apply the distributive property (FOIL) to check your answer

✓

Do NOT PRINT:

Example 2:

a) What is the GCF of $-6x^3y$, $12x^2y$, $3xy$?

b) Factor $-6x^3y - 12x^2y + 3xy$.

Procedure

(i) Select the tiles which represent the product, i.e., area.

(ii) Make a rectangular array of the tiles by placing large square tiles in the upper left corner and the unit (1) tiles in the lower right corner.

(iii) Read the dimensions, i.e., factors, of the completed rectangle.

Example 1: Factor $(x^2 + 5x + 6)$.