2 Rational Expressions part 2

November 30, 2021   7:46 PM

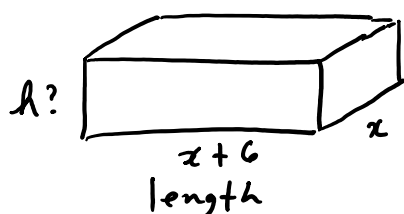## RATIONAL EXPRESSIONS

Rational expressions can be used to model many situations.

Example 1:   The *volume* of a right rectangular prism in cubic centimetres is given by the polynomial $x^3 + 4x^2 - 12x$, where x is the *width* of the prism's base in centimetres. The length of the prism's base is $(x + 6)$ centimetres.

a)   Write a rational expression that will represent the prism's *height*.

b)   Simplify the rational expression. What does this expression represent?

c)   State the permissible values for x.

$$V = x^3 + 4x^2 - 12x$$

Volume = length × width × height

a) $\dfrac{V}{l \times w} = \dfrac{l \times w \times h}{l \times w}$

$$h = \dfrac{V}{l \times w}$$

$$= \dfrac{x^3 + 4x^2 - 12x}{(x+6)x} \quad \text{factor out } x$$

$$= \dfrac{x(x^2 + 4x - 12)}{(x+6)x} \quad \text{factor}$$

$$= \dfrac{x(x+6)(x-2)}{(x+6)x}$$

b) $\boxed{h = x - 2}$

c) non-permissible values

even though it cancelled out! → $x + 6 \neq 0$ and $x \neq 0$

$-6 \; -6$

$\boxed{x \neq -6 \quad \text{and} \quad x \neq 0}$

Example 2: The current of a river is c miles per hour. A boat's speed in still water is 12 miles per hour.

a) Write an expression that will represent the time required for this boat to travel 150 miles downriver.

b) Write an expression that will represent the time required for this boat to travel 150 miles upriver.

c) Write an expression that will represent the difference in the times required for this boat to travel 150 miles downriver and upriver.

$d = $ distance
$s = $ speed
$t = $ time
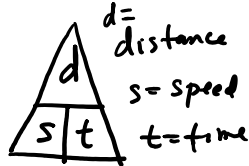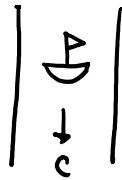
$$d = s \cdot t$$
$$s = \frac{d}{t}$$
$$t = \frac{d}{s}$$

Textbook uses "r" instead of "s" for rate of speed

Speed of boat (mi/h)
- in still water (no current): 12
- with current (downriver): $12 + c$
- against current (upriver): $12 - c$

a) $t_{downriver} = \dfrac{d}{s} = \dfrac{150}{12+c}$

b) $t_{upriver} = \dfrac{d}{s} = \dfrac{150}{12-c}$

c) Difference?

$$t_{upriver} - t_{downriver}$$
$$= \frac{150}{12-c} - \frac{150}{12+c}$$

Assignment: Do "**What kind of food do math teachers eat?**" worksheet *and*
Word Problems: Sec. 6.1 p. 319 #15, 18a, 20a, opt: 21-24, 29a.

Do not print

Solution #1   a) The volume of a right rectangular prism is the product of its base area and its height.

The base area is the product of the base's width and length.

The prism's height must be the result of dividing its volume by the product of the base's width and length.

The prism's height is

$$\frac{x^3 + 4x^2 - 12x}{x(x + 6)}$$

b)   To simplify the numerator must be factored.

$$\frac{x(x^2 + 4x - 12)}{x(x + 6)}$$

$$\frac{x(x + 6)(x - 2)}{x(x + 6)}$$

Divide the numerator and denominator by the common factors.

$$x - 2$$

$x - 2$ represents the prism's height in centimetres.

c)   All the dimensions must be longer than 0 cm. $x > 2$

$$\left[\text{Answer: } \frac{150}{12+c} \text{ hours, } \frac{150}{12-c} \text{ hours, } \left(\frac{150}{12-c} - \frac{150}{12+c}\right) \text{hours}\right]$$