# 3 Counterexamples (1.3)

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## Day 3: Using Reasoning to Find a Counterexample to a Conjecture (1.3)

Counterexample:
an example that disproves a conjecture
↳ invalid! False!

**Example** 1:
Tom and Jerry were given the following number pattern by their teacher.

$2^2+1=5$ (prime number)
$4^2+1=17$ (prime number)
$6^2+1=37$ (prime number)
$10^2+1=101$ (prime number)

a) Tom made the following conclusion:

"When an even number is squared and one is added, the result is a prime number (a natural number that is greater than 1 and can only be divided by 1 and itself)

Tom's conclusion is a conjecture based on evidence (examples)

b) Jerry investigated Tom's conclusion further. Does Jerry agree or disagree with Tom? Explain.

- $8^2+1=65$ (not prime)
- $12^2+1=145$ ( " " )

So Tom's conjecture is False!

**Example 2**:
Bart and Lisa were discussing what conjecture could be made regarding the prime number pattern shown.

3+5=8
5+7=12
7+11=18
13+17=30
19+23=42
29+31=60

Their conjecture: "The sum of two prime numbers is an even number."

Is this a valid conjecture? If not a) provide a *counterexample* and b) *revise* the conjecture so it is true for all prime numbers.

Invalid counterexample: $2+3=5$

$47+2=49$

Revised: The sum of 2 primes (excluding 2) is even.

**Example 3**:

1, 2, 3, ....

Find a counterexample to this conjecture.

"Natural numbers can be written as the sum of consecutive numbers." Here are some examples:

4+5=9

3+4+5=12

4+5+6+7=22

counter-example: 1+2 = 3, ~~1+1 = 2~~, ~~2+2 = 4~~

**Example 4**:

Here is an interesting numeric pattern:

| | A | B |
|---|---|---|
| 1 | 1(8)+1 | 9 |
| 2 | 12(8)+2 | 98 |
| 3 | 123(8)+3 | 987 |
| 4 | 1234(8)+4 | 9876 |
| 5 | 12345(8)+5 | 98765 |

Find a counterexample that breaks the pattern.

6 — 123456(8)+6 — 9876545

7 — 1234567(8)+7 — 98765434

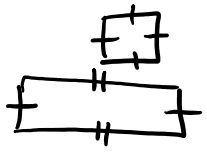
8 ✓

9 ✓

10 → 12345678910(8)+10 — 9876543210

**Example 5**:
Are the following conjectures valid? If it is not, find a **counterexample**.

a) Every rectangle is a square.

F 

b) Every square is a rectangle.

→ 4 sides & 4 right ∠'s

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c) If two angles are acute, their sum is less than 180°.

↳ Let A < 90° T A + B < 180°
B < 90° ∠90° + ∠90° < 180°

d) The square of a number is always greater than the number.

Try $(-5)^2 = 25$ ✓ $(0.5)^2 = 0.25$

$\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$

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e) If one is added to an odd number, then the result is always an even number.

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$1 + 1 = 2$
$1 + 3 = 4$
$1 + 5 = 6$

f) A number that is not positive is negative.

F 0 is neither pos. nor. neg!

In summary:
1. You need only  counterexample to disprove a conjecture!

**Assignment**: Sec 1.3, p. 22 #1 (pick any 3). Pick any 4 out of: 3-6, 12, 14. Do #10 (how is #10 different from the other questions?)