3 Dividing Radicals

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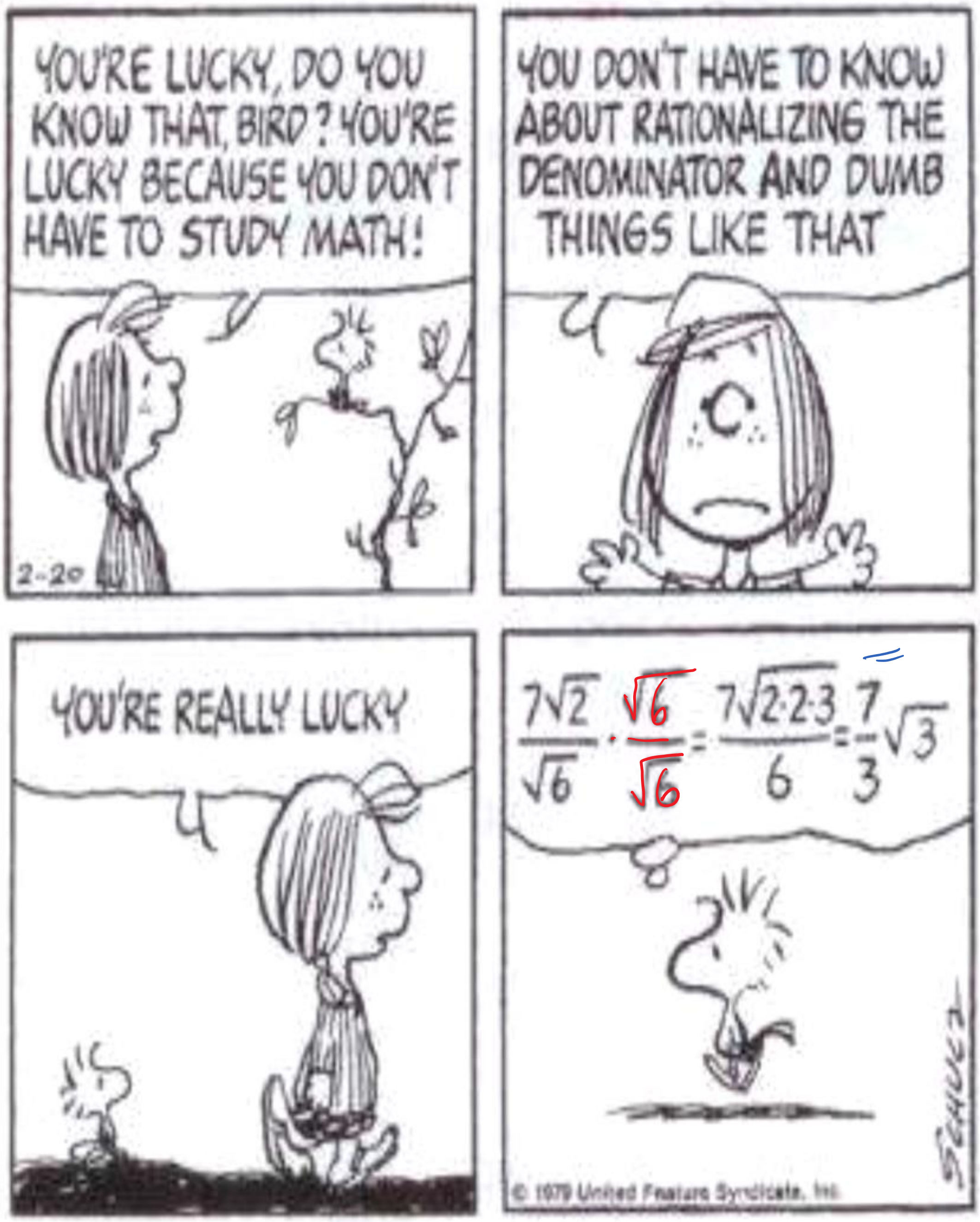

$= \frac{7 \cdot \cancel{2}\sqrt{3}}{\underset{3}{\cancel{6}}}$

SIMPLIFYING RADICALS $\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$ $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

The three conditions for simplest radical form are:

- the radicand cannot have a factor that is perfect square larger than 1.
- the radicand cannot be a fraction or decimal, $\cancel{\sqrt{\frac{5}{6}}}$ $\cancel{\sqrt{1.5}}$ → twins escape!
- the denominator cannot contain a radical!

Example 1: Simplify

a) $\frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{30}{6}} = \boxed{\sqrt{5}}$

b) $\sqrt{\frac{18}{75}}\begin{matrix}\div 3\\ \div 3\end{matrix} = \sqrt{\frac{6}{25}} = \frac{\sqrt{6}}{\sqrt{25}} = \frac{\sqrt{6}}{\sqrt{5\cdot5}} = \boxed{\frac{\sqrt{6}}{5}}$

An **irrational number** is non-repeating and non-terminating. (A famous irrational number is pi.)

A SLICE OF PI
****
3.14159265358979
323846264338327
95028841971693
9937510582097
494459230781
64062862089
9862803482
534211706
79821480
8651328
230664
70938
4460
955
05
8

When the denominator of a fraction is an irrational number, the denominator goes on forever, so it is *impossible* to divide by such a decimal number! The process or rewriting a fraction so that the denominator is not irrational is called ***rationalizing the denominator***.

Case 1: Rationalize the Denominator where denominator is a MONOMIAL

Example 2: Simplify

a) $\frac{\sqrt{5}}{\sqrt{7}}\boxed{\frac{\sqrt{7}}{\sqrt{7}}} = \frac{\sqrt{5\cdot7}}{\sqrt{7\cdot7}} = \boxed{\frac{\sqrt{35}}{7}}$

c) $\frac{1}{\sqrt[3]{5}}\ \frac{\sqrt[3]{5}}{\sqrt[3]{5}}\ \frac{\sqrt[3]{5}}{\sqrt[3]{5}}$

$= \frac{\sqrt[3]{5\cdot5}}{\sqrt[3]{5\cdot5\cdot5}}$

$= \boxed{\frac{\sqrt[3]{25}}{5}}$

Multiply by enough radical siblings to escape!

b) $\frac{15}{\sqrt{20}} = \frac{15}{\sqrt{2\cdot2\cdot5}} = \frac{15}{2\sqrt{5}}\ \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{15\sqrt{5}}{2\sqrt{5\cdot5}}$

$= \frac{\overset{3}{\cancel{15}}\sqrt{5}}{2\cdot\cancel{5}}$

$= \boxed{\frac{3\sqrt{5}}{2}}$

20
4 5
2 2

Case 2: Rationalize the Denominator where denominator is a BINOMIAL

CONJUGATES

- **conjugate** of **(a+b)** is **(a-b)**   Ex: conjugate of $1+\sqrt{5}$ is $1-\sqrt{5}$
- **conjugate** of **(a-b)** is **(a+b)**
- The product of conjugates $\boxed{(a+b)(a-b) \text{ is } a^2-b^2.}$

↑ shortcut instead of FOILing!

	a	b
a	a^2	ab
-b	-ab	$-b^2$

Example 3: Multiply $\left(7\sqrt{3}-2\sqrt{5}\right)$ by its conjugate.

$$(a \quad - \quad b)(a \quad + b)$$

$$\left(7\sqrt{3}-2\sqrt{5}\right)\left(7\sqrt{3}+2\sqrt{5}\right)$$

$$= a^2 - b^2$$

$$\left(7\sqrt{3}\right)^2 - \left(2\sqrt{5}\right)^2$$

$$7^2(\sqrt{3})^2 - 2^2(\sqrt{5})^2$$

$$= 49\cdot 3 - 4\cdot 5$$

$$= 147 - 20$$

$$= \boxed{127}$$

Note that the product of binomial radical conjugates is a rational number.

DIVIDING RADICAL EXPRESSIONS

Consider the division $6 \div \left(1+\sqrt{5}\right)$; this division results in the quotient $\dfrac{6}{1+\sqrt{5}}$

This fraction is not in simplest form because there is a radical in the denominator.

Example 4: Simplify $\dfrac{6}{1+\sqrt{5}}$

- Use the conjugate to simplify

$$\frac{6}{(1+\sqrt{5})} \cdot \boxed{\frac{(1-\sqrt{5})}{(1-\sqrt{5})}}$$

$$= \frac{6 - 6\sqrt{5}}{1^2 - (\sqrt{5})^2}$$ use shortcut! a^2-b^2

$$= \frac{6-6\sqrt{5}}{1-5} = \frac{6-6\sqrt{5}}{-4}$$

$$= \frac{\cancel{6}^{3}}{\cancel{-4}_{-2}} - \frac{\cancel{6}^{3}\sqrt{5}}{\cancel{-4}_{2}}$$

$$\frac{3-3\sqrt{5}}{-2}$$

weird to leave negative in denominator!

$\boxed{\text{Note: } \frac{2}{-3} = \frac{-2}{3} = -\frac{2}{3}}$

$$= \frac{(-1)(3-3\sqrt{5})}{(-1)(-2)}$$

$$= \boxed{\frac{-3+3\sqrt{5}}{2}} = \boxed{\frac{-3(1-\sqrt{5})}{2}}$$

Assignment: Sec. 5.2, p. 290 # 6ad, 7a, 8ac, 9a, 10bcd, 13. Extra practice (opt): 11, 17, 19-20.

exercise: Simplify:

a) $\dfrac{5\sqrt{6}}{2\sqrt{11}}$ b) $\dfrac{5\sqrt{7}}{2\sqrt{90}}$ c) $\dfrac{5}{\sqrt[3]{4}}$

exercise: Simplify $\dfrac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}$ (Hint: multiply up & down by conjugate of denominator!)