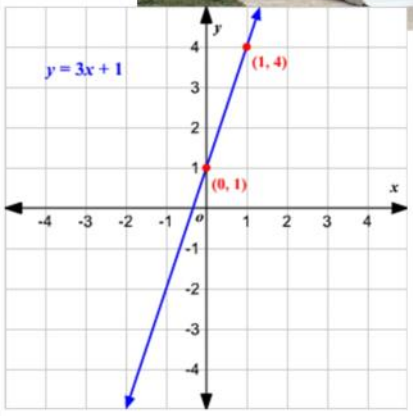
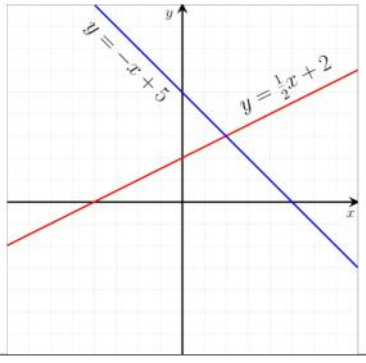
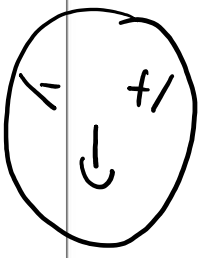
3 Exploring Quadratic Functions: Effects of a, b, c in standard form

March 31, 2021 6:20 PM

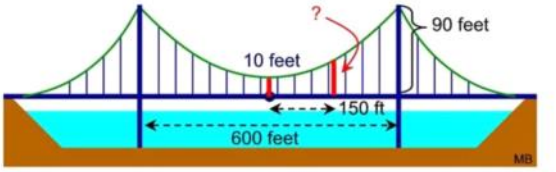
What do you notice? What to you wonder?

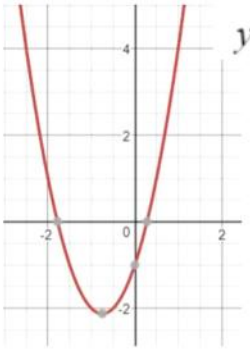




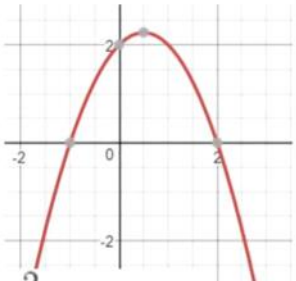


Mr. Slope Guy





$y = 2x^2 + 3x - 2$



$y = -x^2 + x + 2$

Observations

Graph: Line

Equation: $y = mx + b$

slope y-intercept

Graph: quadratic, parabola

Standard form: $y = ax^2 + bx + c$

Standard Form of a Quadratic Function

$$y = ax^2 + bx + c$$

where *x* is the input variable; *y* is the output variable; *a* and *b* are coefficients $(a \neq 0)$, and *c* is a constant.

How do the values *a*, *b*, and *c* affect the parabola?!

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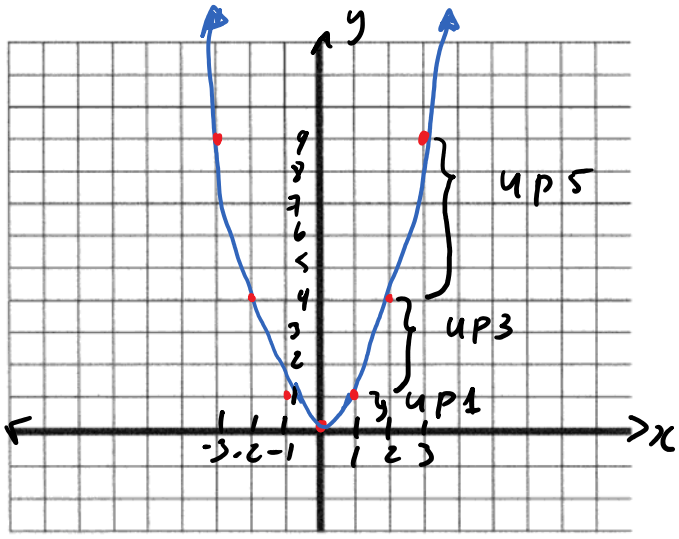
U02A12

THE BASIC PARABOLA

Equation	$y = x^2$

Table of Values

x	y
-3	$(-3)^2 = 9$
-2	$(-2)^2 = 4$
-1	1
0	0
1	1
2	4
3	9



Fill in the following information about the parabola:

What is the vertex?	What is the Direction of Opening?	What's the "step pattern" of the parabola? (how do you move from point to point, starting from the vertex? – and it doesn't matter if you go to the right or left)		
(0,0)	up ↑		Over 1 →	1↑
			Over 1 →	3↑
			Over 1 →	5↑

Since all parabolas have their "over" steps the same, we usually refer to these three numbers as the Step Pattern of the parabola
So, the Step Pattern of this parabola is 1, 3, 5, 7, ... ↑

always go over by 1

$y = ax^2 + bx + c$
here, $a = 1$, $c = 2$

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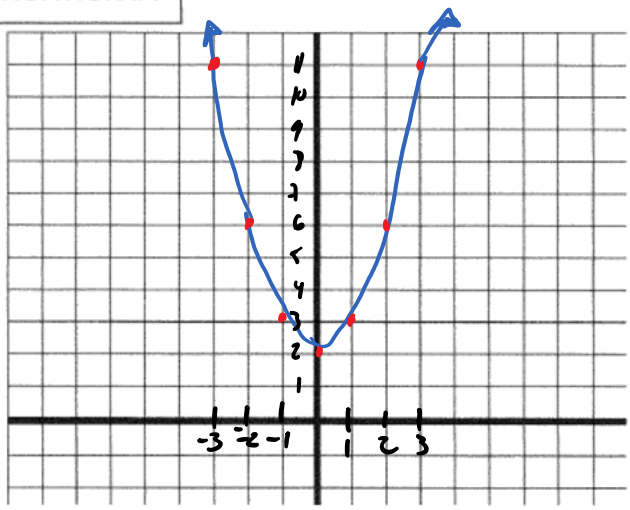
PARABOLA INVESTIGATION #1

Equation	$y = x^2 + 2$

Table of Values

x	y
-3	$(-3)^2 + 2 = 9 + 2 = 11$
-2	6
-1	3
0	2
1	3
2	6
3	11



Fill in the following information about the parabola:

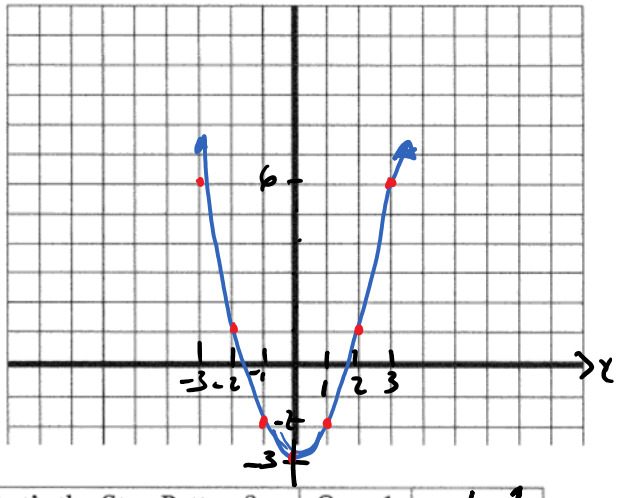
What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,2)	UP		Over 1	1
			Over 1	3
			Over 1	5

PARABOLA INVESTIGATION #2

Equation	$y = x^2 - 3$

Table of Values

x	y
-3	$(-3)^2 - 3 = 9 - 3 = 6$
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



Fill in the following information about the parabola:

What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,-3)	UP		Over 1	1
			Over 1	3
			Over 1	5

Observations:

The value of *c* is equal to the y-intercept.

It moves the parabola up or down c units.

1:50

$y = ax^2 + bx + c$

Here, $a = 2$

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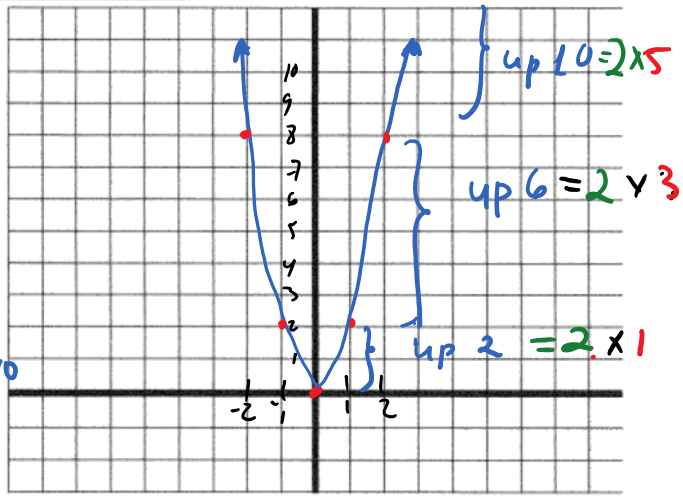
PARABOLA INVESTIGATION # 5

Equation	$y = 2x^2$
	$a = 2$

Table of Values

x	y
-3	$2(-3)^2 = 2(9) = 18$
-2	8
-1	2
0	0
1	2
2	8
3	18

+2, +6, +10



Fill in the following information about the parabola:

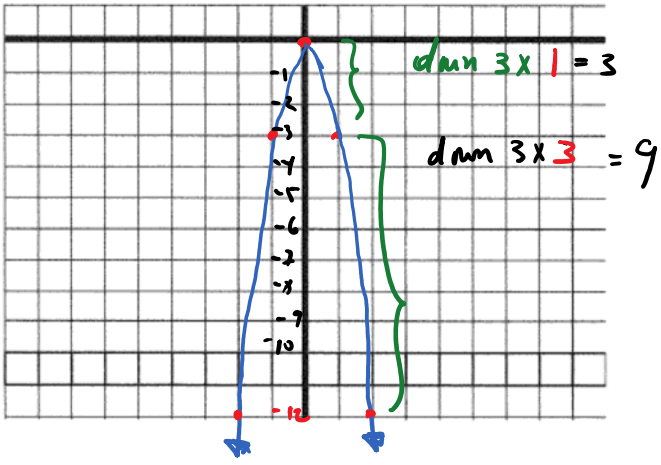
What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,0)	UP	$2 \times 1, 2 \times 3, 2 \times 5$ → 2, 6, 10	Over 1	2
			Over 1	6
			Over 1	10

PARABOLA INVESTIGATION # 6

Equation	$y = -3x^2$
	$a = -3$

Table of Values

x	y
-3	-27
-2	-12
-1	-3
0	0
1	-3
2	-12
3	-27



Fill in the following information about the parabola:

What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,0)	DOWN	$-3 \times 1, -3 \times 3, -3 \times 5$ → 3, -9, -15	Over 1	3
			Over 1	9
			Over 1	15

★ negative means move downwards

Observations

If a is positive, then parabola opens UP

If a is negative, then parabola opens DOWN

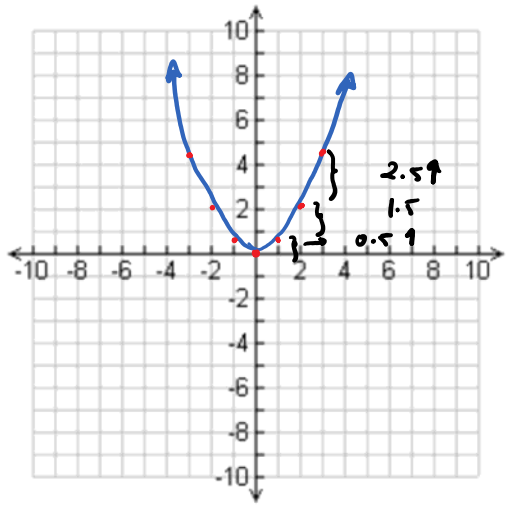
Parabola Investigation (where *a* is a fraction)

Equation:

$$y = \frac{1}{2}x^2$$

Table of Values:

x	*y*
−3	$\frac{1}{2}(-3)^2 = \frac{1}{2}(9) = 4.5$
−2	2
−1	0.5
0	0
1	0.5
2	2
3	4.5



What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,0)	up	$\frac{1}{2}\times 1$, $\frac{1}{2}\times 3$, $\frac{1}{2}\times 5$ → 0.5, 1.5, 2.5,...	Over 1	0.5↑
			Over 1	1.5↑
			Over 1	2.5↑

Observations

If *a* is fraction between −1 & 1, then parabola is wider

If *a* is >1 / <−1, then parabola is thinner, narrow

Example 1: Find the *y*-intercept of $y = -3x^2 + 12x - 10$

The y-intercept is −10.

The parabola opens up/down because a is neg (−3)

The parabola is narrow/wide because a is not a fraction between −1 & 1 (or a>1)

Summary:

a,b,c	Effect on $y = ax^2 + bx + c$	Example/Diagram
a is positive	opens up	
a is negative	opens DOWN	
a > 1 a < −1	Parabola is narrow	
−1 < a < 1 (Between −1 & 1)	Parabola is wide	
c	y-intercept	c → 

step pattern: ↑ a ×1, a ×3, a ×5, ...
If a is neg, move ↓
Always move over → by 1

Example 2: Complete the table.

	Quadratic Function	Value of *a* (coefficient	Direction of opening	*y*-intercept	Step Pattern (e.g. 1, 3, 5, ...)

Always move over by 1

Example 2: Complete the table.

	Quadratic Function	Value of a *(coefficient of x^2)*	Direction of opening (Up/Down)	y-intercept	Step Pattern (e.g., 1, 3, 5, …)
a	$y = 1x^2 - 4x + 5$	1	↑	5	1, 3, 5
b	$y = -1x^2 - 8x - 10$	-1	↓	-10	-1, -3, -5
c	$y = 4x^2 - 32x + 63$ narrow	4	↑	63	4, 12, 20 64 4x1, 4x3, 4x5, …
~~d~~	~~$y = 3(x+7)^2 - 4$~~ Hint: Expand!				
~~e~~	~~$y = -2(x-10)^2 + 3$~~				

Example 3: Graph parabolas with:

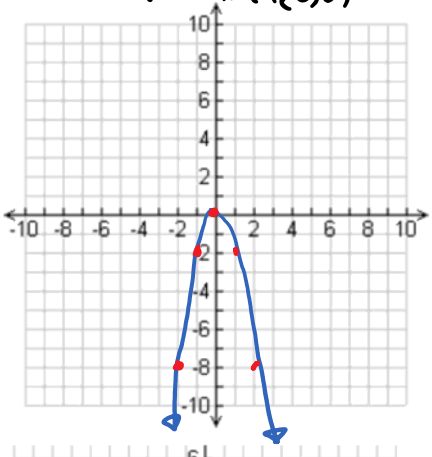
a) $y = -2\boxed{x^2}$ b) Vertex at (3, 4); $a = -1$ c) vertex at (-2, -1), $a = 2$ d) $y = \frac{-1}{2}x^2$

a = -2

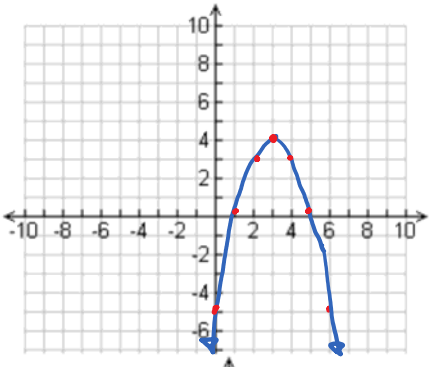
vertex at (0,0)

-2 x 1 = -2
-2 x 3 = -6
-2 x 5 = -10

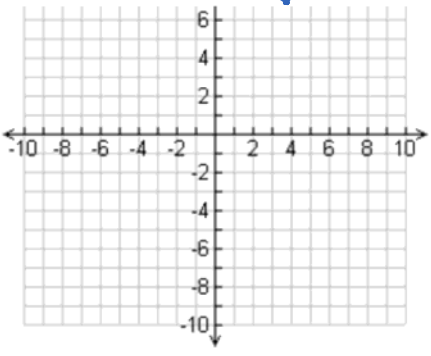
a)



b)



c)



d)