3 Linear Inequalities in 2 Variables

November 26, 2019 9:43 PM

INEQUALITIES

An inequality is a mathematical statement that compares values that may not be equal.

- $<$ is the symbol for "is less than" $8 < 12$
- $>$ is the symbol for "is greater than" $-8 > -12$
- $\leq$ is the symbol for "is less than or equal to"
- $\geq$ is the symbol for "is greater than or equal to"

Investigate:

Write an inequality, e.g., 3 < 10 or 59 > -16: $3 < 10$

Multiply both sides by a negative number: $(-1)\,3 < (-1)\,10$

Did you need to change anything? If so, what? $-3 > -10$

If you × or ÷ by a negative, change "<" to ">", etc.

The same rules for equations can be applied to inequalities with one exception!

When multiplying or dividing both sides of an inequality by negative number, the direction of the inequality symbol must be reversed.

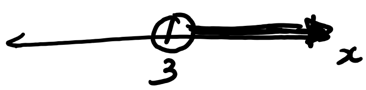
To solve any inequality, find all the values of the variable that *satisfies the inequality*.

Example 1: Solve $7 - 2x < 1$ and graph its solution set.

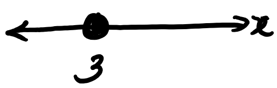
$$-2x < -6$$

$$x > 3$$

- Its graph is on a number line.



vs. $x \geq 3$



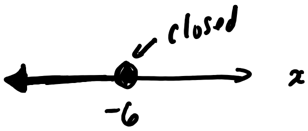
Example 2: Solve $5 - 3x \geq 23$ and graph its solution set.

$-5 \quad -5$

$-3x \geq 18$

$\overline{-3} \quad \overline{-3}$

$x \leq -6$



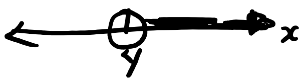
Example 3: Solve $3x - 20 > -2x$ and graph its solution set.

$+2x \; +20 \quad +2x \; +20$

$5x > 20$

$\overline{5} \quad \overline{5}$

$x > 4$



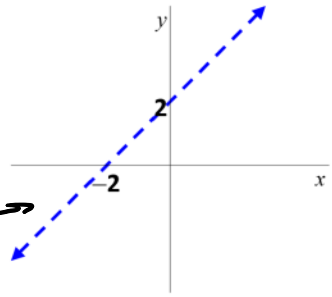
If $>$ or $<$, use open circle

If $\geq$ or $\leq$, " closed "

LINEAR INEQUALITIES IN TWO VARIABLES

To graph the solution of a linear inequality in 2 variables:

- Draw the boundary line:
  - Change the inequality to "=" and graph that line.
  - Use a solid line if points on the boundary satisfy the inequality (i.e., ≤ or **≥**).
    Use a **dashed**/**broken line** if points on the boundary do not satisfy the inequality (i.e., **<** or **>**).



- Solution region: Determine the region with the points that satisfy the inequality.
  - Choose a point on one side of the boundary and check if its coordinates satisfies the inequality. **Trick**: (0,0) is an easy point to test!
  - If the point satisfies the inequality (i.e, is TRUE), shade that region; otherwise, shade the *other* region.

use dashed line for boundary

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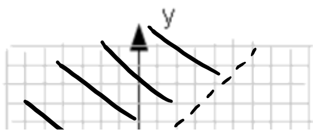
Example 4: Draw the graph of $y > x + 2$.



- Change inequality to '='. $y = x + 2$ y-int

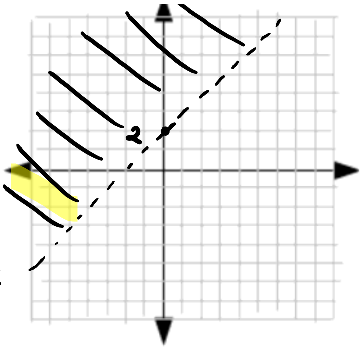
- Change inequality to '='.
- Graph the boundary line.

$y = x + 2$ (y-int)
$y = mx + b$
$m = \frac{1}{1} = \frac{\uparrow 1}{\to 1}$

- Using the inequality, test a point that's not on the line. Trick: Test (0,0)!

$y > x + 2$
$0 > 0 + 2$
$0 > 2$ FALSE → shade other side

- Solution region: If inequality is TRUE, shade side with the point tested. If FALSE, shade the other side!



For any inequality statement that is solved for y, the solution will include:

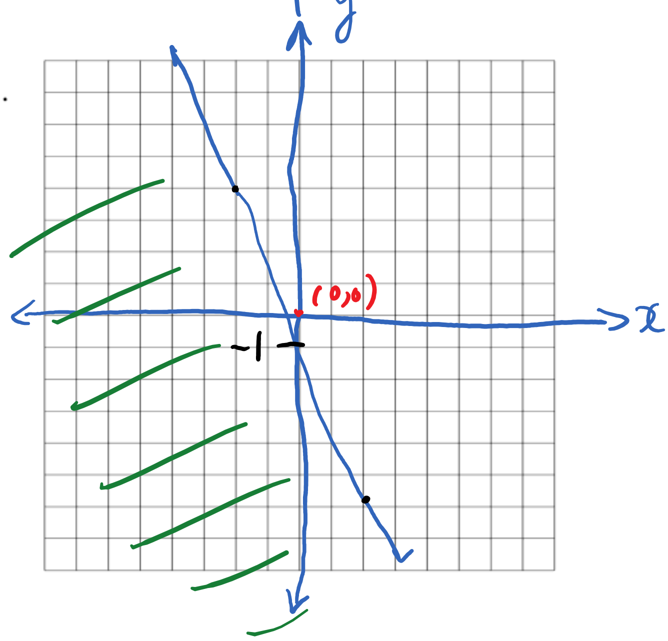
- ➢ points **above** the boundary line for $>$ **or** $\geq$ inequalities,
- ➢ points **below** the boundary line for $<$ **or** $\leq$ inequalities,

Example 5: Draw the graph of $y \leq -\frac{5}{2}x - 1$. (solid)

y-int: $b = -1$

$m = \frac{-5}{2} = \frac{\downarrow 5}{\to 2}$

Test (0,0): $0 \leq -1$ FALSE → shade other side



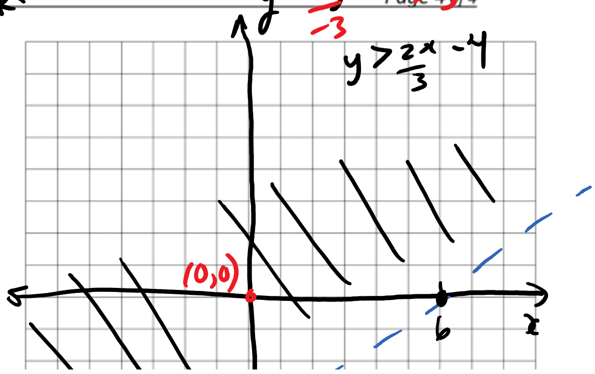
Example 6: Draw the graph of $2x - 3y < 12$. (dashed) OR Rewrite

$2x - 3y < 12$
$-2x \quad -2x$
$-3y < -2x + 12$
$\div -3$
$y > \frac{2x}{3} - 4$

Bob's exercise:
- burns 500 cal/hr jogging, 300 cal/hr walking
- Wants to burn AT LEAST 3000 cal.
- Graph possible combinations.
- Pick 1 and check that it satisfied the inequality!

To find x-intercept, let y = 0 + use equals.

$2x - 3(0) = 12$



To find x-intercept, let y = 0 & solve equation.

$2x - 3(0) = 12$

$2x = 12$

$x = 6$

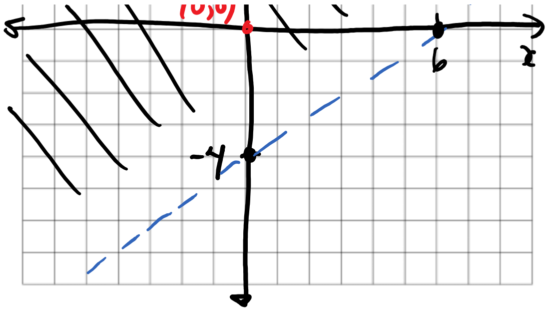
To find y-intercept, let $x = 0$

$2(0) - 3y = 12$

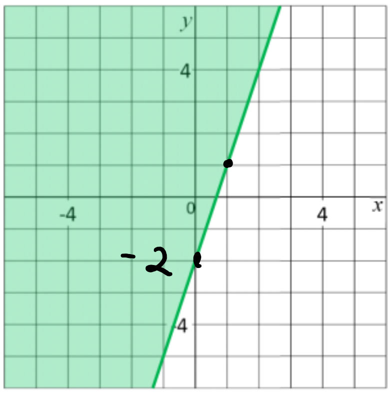
$y = -4$

Test (0,0): $0 - 0 < 12$

$0 < 12$ TRUE



Example 7: Write the inequality for each graph.

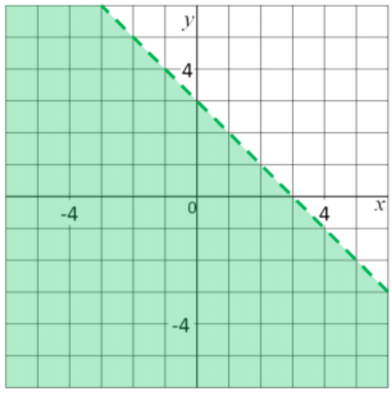


$y = mx + b$

$m = \frac{3}{1} = 3$

$y \geq 3x - 2$

solid line: $\geq$ $\leq$
dashed line: $>$ $<$
shading above: $\geq$ $>$
shading below: $\leq$ $<$



Bob: jogging: 500 cal/hr

walking: 300 cal/hr

at least → $\geq$ 3000 cal

Let x = jogging in hours

y = walking in hours

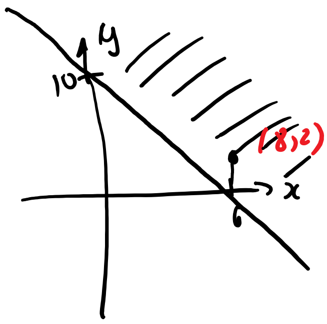
$500x + 300y \geq 3000$

$x=0$: $300y = 3000$

$y = 10$

$y=0$: $500x = 3000$

$x = 6$



one possibility is 8 hours of jogging & 2 " " walking

Assignment: Sec 9.1, p. 472 #1-ac, 3-4ace, 8abc (graph by hand), 9, 13, 15