3 Quadratic Equalities in 1 Variable

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- A quadratic equation in one variable: standard form $ax^2 + bx + c = 0$.
- A quadratic **inequality** in one variable will have an inequality symbol (<,≤,>,≥) instead of = .
- A quadratic inequality in one variable can be solved graphically and algebraically.

SOLVING QUADRATIC INEQUALITIES IN ONE VARIABLE GRAPHICALLY

Example 1: Consider the quadratic function $y = x^2 - 6x + 5$

1) Convert to vertex form:

$y = \boxed{x^2 - 6x} + 5$

$\frac{b}{2} = \frac{-6}{2} = -3 \rightarrow (-3)^2 = 9$

$y = \boxed{x^2 - 6x} + 9 - 9 + 5$

$y = (x-3)^2 - 4$

vertex: $(3, -4)$

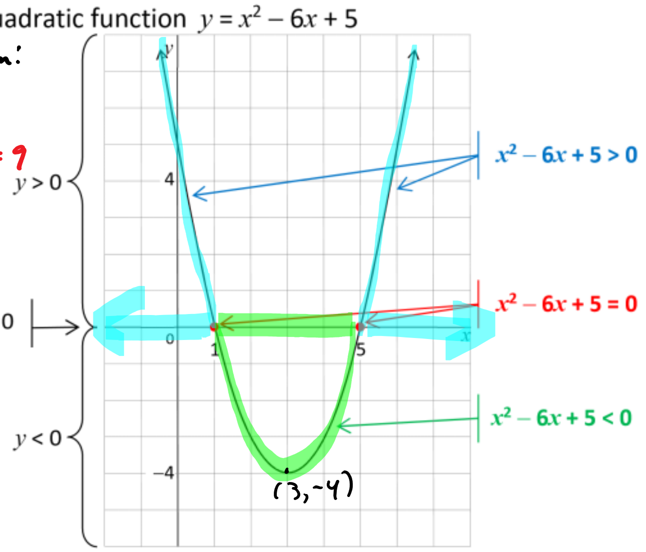
2) Roots are: 1 + 5

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x - 1 = 0$ or $x - 5 = 0$

$x = 1$ or $x = 5$



3) • Solution to "≥" or ">" is found ABOVE the x-axis where $y > 0$

• " " "≤" or "<" " " BELOW the x-axis where $y < 0$

• " " "=" " " on x-axis where $y = 0$

Solve each of the following:

a) $x^2 - 6x + 5 = 0$ factor (see above!)

$\{1, 5\}$

b) $x^2 - 6x + 5 > 0$ use open circle

$\{x \mid x < 1 \text{ or } x > 5, x \in \mathbb{R}\}$ where element of

c) $x^2 - 6x + 5 \geq 0$ use closed circle!

$\{x \mid x \leq 1 \text{ or } x \geq 5, x \in \mathbb{R}\}$

d) $x^2 - 6x + 5 < 0$

$\{x \mid 1 < x < 5, x \in \mathbb{R}\}$ Roots

e) $x^2 - 6x + 5 \leq 0$

$\{x \mid 1 \leq x \leq 5, x \in \mathbb{R}\}$

SOLVING QUADRATIC INEQUALITIES IN ONE VARIABLE ALGEBRAICALLY

Example 2: Solve the quadratic inequality $x^2 - 6x + 5 < 0$ open circles

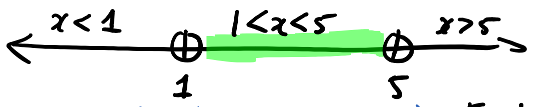
- **Find the Roots**: Solve $x^2 - 6x + 5 = 0$ algebraically: the roots are 1 and 5.

Example 2: Solve the quadratic inequality $x^2 - 6x + 5 < 0$

- **Find the Roots**: Solve $x^2 - 6x + 5 = 0$ algebraically; the roots are 1 and 5. Place the roots on a number line; use **closed circles** if these numbers are included in the solution and **open circles** if these numbers do not satisfy the inequality.

- **Roots and Test Points**: These numbers break up the number line into regions. Test a value from within each region; if it satisfies the inequality, then all the numbers from that region will satisfy it as well.

$x^2 - 6x + 5 < 0$



Test each region!

Test 0:
$x^2 - 6x + 5 < 0$
$0^2 - 6 \cdot 0 + 5 < 0$
$5 < 0$
FALSE

Test 2
$2^2 - 6(2) + 5 < 0$
$4 - 12 + 5 < 0$
$-3 < 0$
TRUE! ∴ shade this region

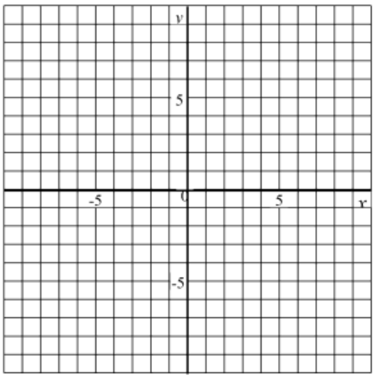
Test 6
$\cancel{6^2} - \cancel{6 \cdot 6} + 5 < 0$
$5 < 0$
FALSE
∴ not part of solution

$\boxed{\{x \mid 1 < x < 5, x \in \mathbb{R}\}}$

Write the solution set by describing the x-values from all the regions that satisfy the quadratic inequality.

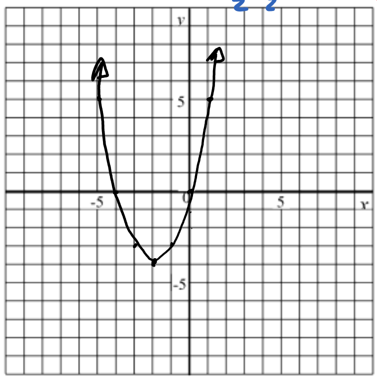
Example 3: Solve graphically.

a) $x^2 \leq 9$



b) $x^2 + 4x > 0$

$\frac{b}{2} = \frac{4}{2} = 2 \rightarrow (2)^2 = 4$



$y = \underbrace{x^2 + 4x + 4} - 4$

$y = (x+2)^2 - 4$

Vertex: $(-2, -4)$

Roots?

$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0$ or $x + 4 = 0$

$x = -4$

open circle

Test: -6	Test -1	Test 1
$x^2 + 4x > 0$	$x^2 + 4x > 0$	$x^2 + 4x > 0$
$(-6)^2 + 4(-6) > 0$	$(-1)^2 + 4(-1) > 0$	$1^2 + 4(1) > 0$
$36 - 24 > 0$	$1 - 4 > 0$	$1 + 4 > 0$
$12 > 0$	$-3 > 0$	$5 > 0$
TRUE	FALSE	TRUE
∴ shade!	∴ don't shade	∴ shade!

$\boxed{\{x \mid x < -4 \text{ or } x > 0, x \in \mathbb{R}\}}$

Assignment: Sec 9.2, p. 486 #1-3, 4, 7-9ac, 13, 15.

exercise: Solve algebraically.

a) $x^2 - 16x + 63 \geq 0$

b) $x^2 + 2x - 1 < 0$

c) $x^2 > 0$

d) $x^2 + 4x + 5 < 0$