3 Standard Deviation

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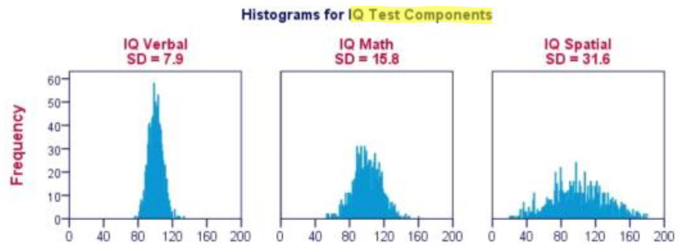
FOM 11 **5.3 Standard Deviation**



Deviate: to vary/move away from the Mean

Deviation: the difference between a data value, x, and the mean, $\bar{x}$, of the data set.

To describe data numerically, we often use two numbers:

1. **Mean**: the average

   Let $x_1, x_2, x_3, \ldots, x_n$ represent any set of values.

   Sum (upper sigma)

   Mean: $\bar{x} = \mu = \dfrac{\sum x_i}{n} = \dfrac{x_1+x_2+x_3+\ldots+x_n}{n}$

   "mu"

2. **Standard Deviation:** a measure of the extent to which the data cluster around the mean

   Let $x_1, x_2, x_3, \ldots, x_n$ represent any set of values.

   Standard Deviation: $\sigma = \sqrt{\dfrac{\sum (x_i-\bar{x})^2}{n}} = \sqrt{\dfrac{(x_1-\bar{x})^2+(x_2-\bar{x})^2+\cdots(x_n-\bar{x})^2}{n}}$

   lower case sigma

The smaller the standard deviation, the more consistent the results and the closer the data to the mean.

Example 1: Calculate the standard deviation from the following sets of values:

a. 7, 8, 9, 10, 11

mean = $\bar{x} = \mu = \frac{7+8+9+10+11}{5} = \frac{45}{5} = 9$

$n = 5 =$ # of numbers

$$\sigma = \sqrt{\frac{(7-9)^2+(8-9)^2+(9-9)^2+(10-9)^2+(11-9)^2}{5}}$$

$$= \sqrt{\frac{(-2)^2+(-1)^2+0^2+1^2+2^2}{5}}$$

$(-2)^2 = 4$ vs. $-2^2 = -4$

$$= \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2} = 1.41$$

$$\boxed{\sigma = 1.41}$$

← 2 decimal places!

"raw data"

b. 7, 9, 11, 13, 15

mean $= \bar{x} = \mu = \frac{7+9+11+13+15}{5} = 11$

$$\sigma = \sqrt{\frac{(7-11)^2+(9-11)^2+(11-11)^2+(13-11)^2+(15-11)^2}{5}}$$

$$= \sqrt{\frac{40}{5}} = \sqrt{8}$$

$$\boxed{\sigma = 2.83}$$

The more consistent data set is because has SMALLER standard deviation!

Example 2 GROUPED DATA: Calculate the standard deviation for the following sets of data:

a.

Midpoint	Daily Commute Time (min)	Number of Employees (frequency, f)
5	0-10	4
15	10-20	9
25	20-30	6
35	30-40	4
45	40-50	2

1st; 1st; $\frac{10+20}{2} = \frac{30}{2} = 15$

$n =$ Total $= 25$

$$\text{mean} = \bar{x} = \mu = \frac{\sum (f)(\text{midpoint})}{n}$$

$$= \frac{4(5)+9(15)+6(25)+4(35)+2(45)}{25}$$

$$= \frac{535}{25}$$

$$\boxed{\bar{x} = 21.4}$$

frequency, midpoint, mean

$$\boxed{\sigma = \sqrt{\frac{\sum (f)(x-\bar{x})^2}{n}}}$$

$$= \sqrt{\frac{4(5-21.4)^2+9(15-21.4)^2+6(25-21.4)^2+4(35-21.4)^2+2(45-21.4)^2}{25}}$$

$$= \sqrt{\frac{3376}{25}}$$

$$\boxed{\sigma = 11.62}$$

b.

midpoint	# of Orders	# of Days, f
11	10-12	4
14	13-15	12
17	16-18	20
20	19-21	14

Total: 50

$$\text{mean} = \bar{x} = \mu = \frac{4(11)+12(14)+20(17)+14(20)}{50}$$

$$\boxed{\bar{x} = 16.64}$$

$$\sigma = \sqrt{\frac{4(11-16.64)^2+12(14-16.64)^2+20(17-16.64)^2+14(20-16.64)^2}{50}}$$

$$= \sqrt{7.43}$$

$$\boxed{\sigma = 2.72}$$

more consistent because SMALLER σ!

5.

Assignment: Standard Deviation Worksheet