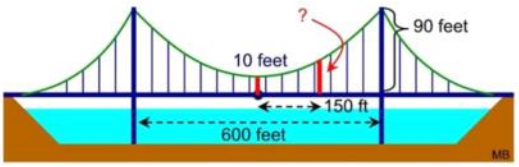
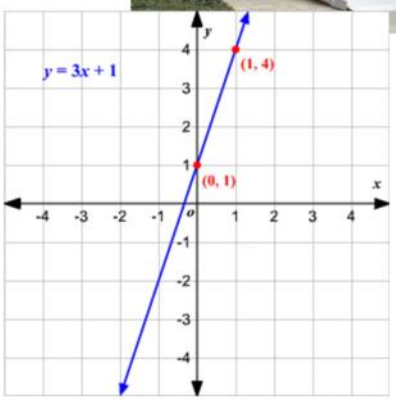
3 Standard Form of Quadratic Functions

September 19, 2021 7:21 PM

PreCal 11 Ch 3 Day 3: Standard Form of Quadratic Functions

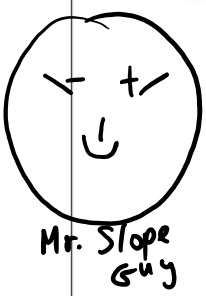
What do you notice? What to you wonder?

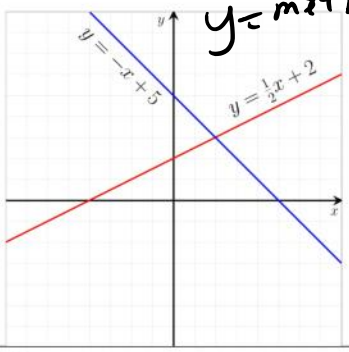


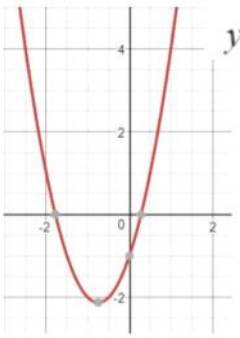


slope

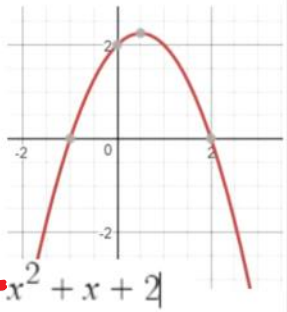
$y = mx + b$







$y = 2x^2 + 3x - 2$



$y = -x^2 + x + 2$

Observations

Graph: Line

Equation: $y = mx + b$

slope

y-intercept

Graph: parabola

Standard form: $y = ax^2 + bx + c$

Standard Form of a Quadratic Function

$$y = ax^2 + bx + c$$

where *x* is the input variable; *y* is the output variable; *a* and *b* are coefficients ($a \neq 0$), and *c* is a constant.

↳ otherwise we get a line!

How do the values *a*, *b*, and *c* affect the parabola?!

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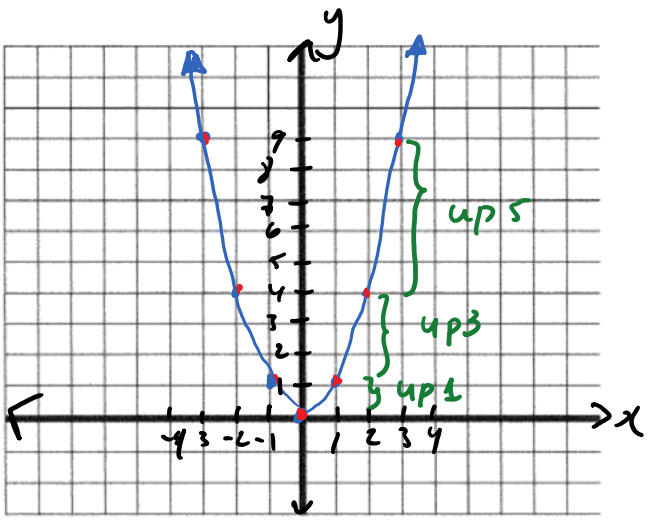
THE BASIC PARABOLA

Equation	$y = x^2$

Table of Values

x	y
-3	$(-3)(-3) = 9$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	0
1	1
2	4
3	9



Fill in the following information about the parabola:

What is the vertex?	What is the Direction of Opening?	What's the "step pattern" of the parabola? (how do you move from point to point, starting from the vertex? – and it doesn't matter if you go to the right or left)		
(0,0)	up		Over 1 →	1↑
			Over 1 →	3↑
			Over 1 →	5↑

"Method of Differences"

Since all parabolas have their "over" steps the same, we usually refer to these three numbers as the "Step Pattern" of the parabola
So, the Step Pattern of this parabola is 1, 3, 5, 7, ...

Domain: $\{x \mid x \in \mathbb{R}\}$ Range: $\{y \mid y \geq 0,\ y \in \mathbb{R}\}$

Now let's look at TRANSFORMATIONS, which change the LOCATION, SHAPE, or ORIENTATION of a graph!

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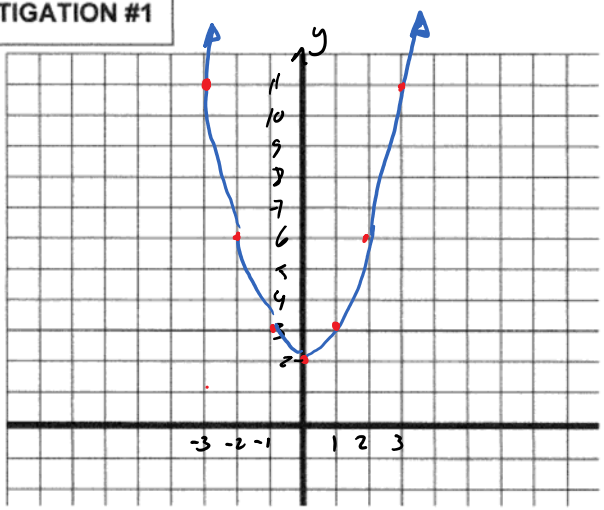
$y = ax^2 + bx + c$

Hint $a = 1, b = 0, c = 2$

PARABOLA INVESTIGATION #1

Equation	$y = x^2 + 2$
Table of Values	
x	$y = x^2 + 2$
-3	$(-3)^2 + 2 = 9 + 2 = 11$
-2	6
-1	3
0	2
1	3
2	6
3	11

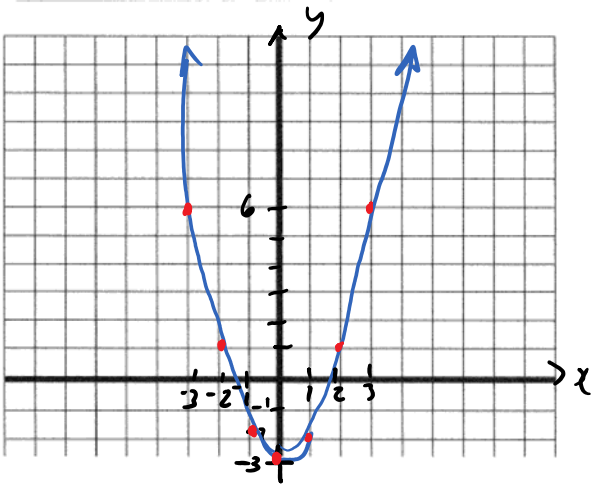


Fill in the following information about the parabola:

What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,2)	UP		Over 1	1 ↑
			Over 1	3 ↑
			Over 1	5 ↑

PARABOLA INVESTIGATION #2

Equation	$y = x^2 - 3$
Table of Values	
x	y
-3	$(-3)^2 - 3 = 6$
-2	1
-1	-2
0	-3
1	-2
2	1
3	6



Fill in the following information about the parabola:

What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,-3)	UP	1, 3, 5, ...	Over 1	1 ↑
			Over 1	3 ↑
			Over 1	5 ↑

Observations:

The value of c is equal to the y-intercept.

It moves the parabola UP or DOWN c units.

$y = ax^2 + bx + c$

Here, $a = 2$, $b = 0$, $c = 0$

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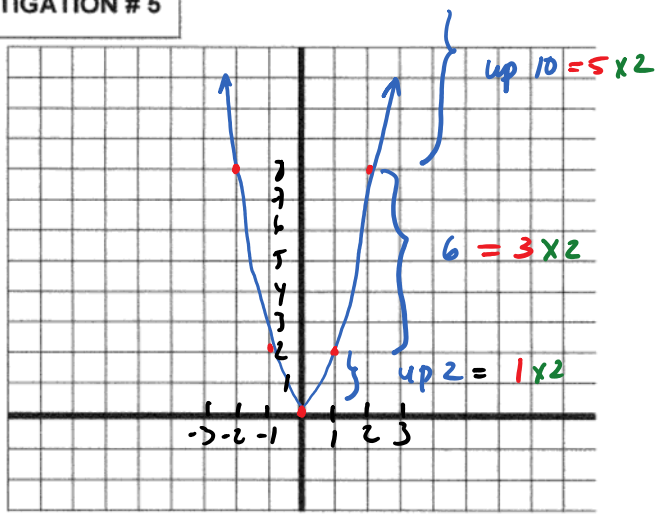
PARABOLA INVESTIGATION # 5

Equation	$y = 2x^2$
	$a = 2$

Table of Values

x	y
-3	$2(-3)^2 = 2(9) = 18$
-2	8
-1	2
0	0
1	2
2	8
3	18
4	$2(4)^2 =$ 32



Fill in the following information about the parabola:

What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,0)	up	2×1, 2×3, 2×5	Over 1	2
		↳ 2, ↳ 6, 6 10	Over 1	6
			Over 1	10

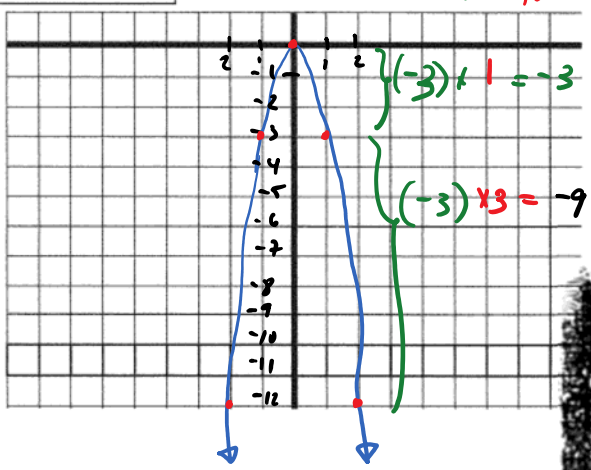
move up $a \times 1$, $a \times 3$, $a \times 5$, etc!

PARABOLA INVESTIGATION # 6

Equation	$y = -3x^2$
	$a = -3$

Table of Values

x	y
-3	-27
-2	-12
-1	-3
0	0
1	-3
2	-12
3	-27



Fill in the following information about the parabola:

What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,0)	Down	-3×1, -3×3, -3×5 ↓	Over 1	-3 ↓
		-3, -9, -15	Over 1	-9 ↓
		↳ Negative move downwards	Over 1	-15 ↓

Observations

If a is positive, then parabola opens up

If a is negative, then parabola open down

→ Reflected about the x-axis.

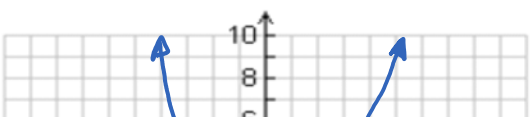
If $a > 1$, $a < 1$, the parabola gets "skinny" (it is vertically expanded/stretched) by a factor of a

Parabola Investigation (where a is a fraction)

Equation:

$$y = \frac{1}{2}x^2 \qquad a = \frac{1}{2}$$

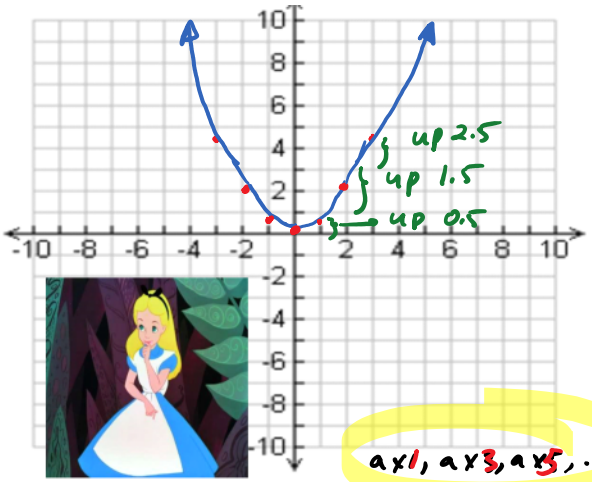
Table of Values:



$y = \frac{1}{2}x^2$ $a = \frac{1}{2}$

Table of Values:

x	y
-3	$\frac{1}{2}(-3)^2 = 4.5$
-2	$\frac{1}{2}(-2)^2 = 2$
-1	0.5
0	0
1	0.5
2	2
3	4.5



a×1, a×3, a×5, ...

What is the vertex?	Direction of Opening?	What's the Step Pattern?		
(0,0)	up	$\frac{1}{2}\times 1, \frac{1}{2}\times 3, \frac{1}{2}\times 5, \ldots$	Over 1	0.5
		$\hookrightarrow 0.5, 1.5, 2.5, \ldots$	Over 1	1.5
		$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$	Over 1	2.5

Observations $-1 < a < 1$

If a is fraction between -1 & 1, then parabola is wider ("vertically compressed")

If a is >1 or <1, then " " narrow ("vertically expanded/stretched")

Example 1: Look at this quadratic function!: $y = -3x^2 + 12x - 10$

The y-intercept is -10.

The parabola opens up/down because a is -3 (negative!)

The parabola is narrow/wide because narrow → vertically expanded/stretched

Summary:

a,b,c	Effect on $y = ax^2 + bx + c$	Example/Diagram
a is positive	up	
a is negative	Down Is reflected about the x-axis	
$a = 1$		no change
$a > 1$ $a < -1$	Parabola is narrow. We say it is vertically expanded/stretched	
-1 1 $-1 < a < 1$	Parabola is wide. We say it is vertically compressed	
c	y-intercept	
Step Pattern	↑ a×1, a×3, a×5, ... ↓ (move downwards if negative!) → always move over 1	

Example 2: Complete the table.

	Quadratic Function	Value of a *(coefficient of x^2)*	Direction of opening (Up/Down)	y-intercept	Step Pattern (e.g., 1, 3, 5, ...)
a	$y = 1x^2 - 4x + 5$	1	∪	5	1, 3, 5, ...
b	$y = -1x^2 - 8x - 10$	-1	∩	-10	-1, -3, -5, ..
c	$y = 4x^2 - 32x + 63$	4	∪	63	4x1, 4x3, 4x5, ... ↳ 4, 12, 20, ..
d	$y = 3(x+7)^2 - 4$ Hint: Expand! $3(x^2+14x+49)-4$ $= 3x^2+42x+147-4$ $= 3x^2+42x+143$	3	∪	143	3x1, 3x3, 3x5 ↳ 3, 9, 15, ...
e	~~$y = -2(x-10)^2+3$~~				

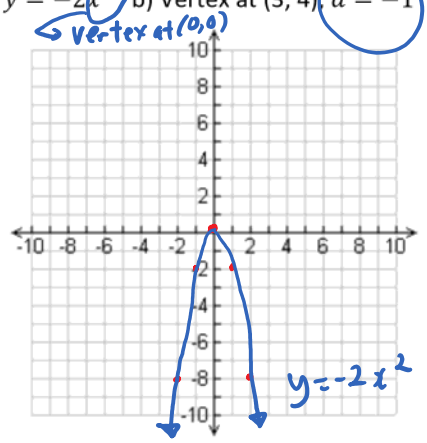
Example 3: Graph parabolas with:

a) $y = -2x^2$ b) Vertex at (3, 4), $a = -1$ c) vertex at (-2, -1), $a = 2$ d) $y = \frac{-1}{2}x^2$

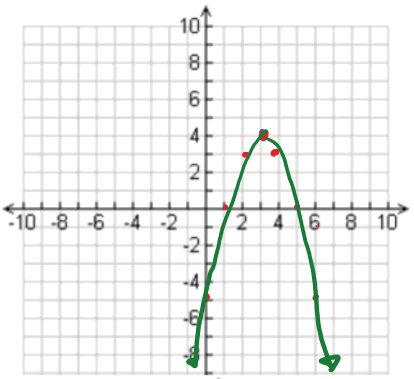
↳ vertex at (0,0)

Steps:
$-2 \times 1 = -2$
$-2 \times 3 = -6$
$2 \times 5 = -10$

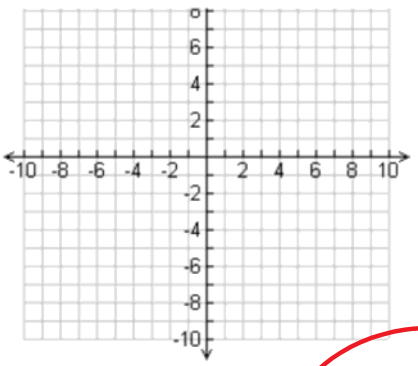
a)



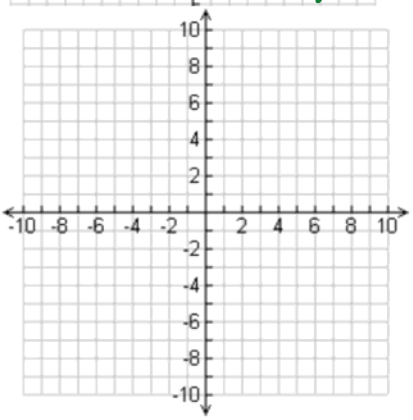
b)



c)



d)



Omit 1e 3e!