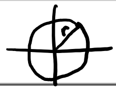
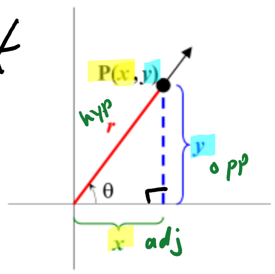
# 3 Trig Ratios of Any Angle Part 1

March 4, 2020 12:17 PM



PRE-CALCULUS 11 Ch 2 – Day 3: TRIGONOMETRIC RATIOS OF ANY ANGLE (Part 1)

## TRIGONOMETRIC RATIOS OF ANGLES IN STANDARD POSITION




Let $P(x,y)$ represent any point on the terminal arm of the standard position angle $\theta$.

Let $r$ represent the distance from the vertex at (0,0) to $P(x,y)$; $r$ is always positive because it is a distance.

$x$, $y$, and $r$ are the lengths of the sides from the right triangle formed in the diagram.

The Pythagorean Theorem gives the relationship between $x$, $y$, and $r$: $\boxed{x^2 + y^2 = r^2}$

The old definitions for the sine, cosine, and tangent of an angle can be applied to angles in standard position using the right triangle in the diagram.

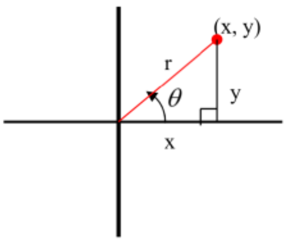
SOH CAH TOA

- The ***sine of angle*** $\theta$, $\sin\theta = \dfrac{y}{r}$
- The ***cosine of angle*** $\theta$, $\cos\theta = \dfrac{x}{r}$
- The ***tangent of angle*** $\theta$, $\tan\theta = \dfrac{y}{x}$

With these definitions, trigonometry is no longer limited to acute angles.
Since $x$ and $y$ can be negative, so can the values of the trigonometric ratios.

<u>**Trig Ratios on the coordinate plane:**</u> Rotating a point around the coordinate plane creates angles in standard position. By dropping a **perpendicular line from the point to the x-axis**, a right triangle is created. Trigonometric ratios occur with respect to the **reference angle.**

<u>**Trig Ratios in Quadrant I:**</u>




$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$
$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$
$$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$$

all positive!

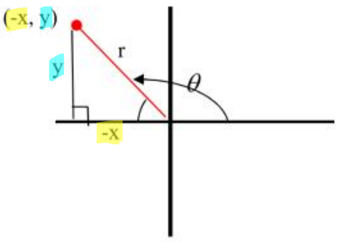
<u>Note:</u>

- "r" is **positive** in all quadrants
- since "x" and "y" are also positive in the first quadrant, all of the **trig ratios** in **quadrant I** are **positive**

The radius is always the hypotenuse!

QII

## Trig Ratios in Quadrant II:




$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$$ } pos.

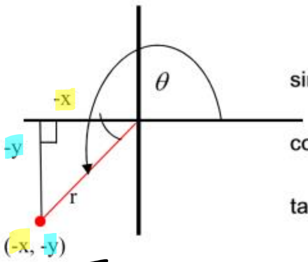
$$\cos\theta = \frac{adj}{hyp} = \frac{-x}{r}$$

$$\tan\theta = \frac{opp}{adj} = \frac{y}{-x}$$

Note:

- "r" is **positive** in all quadrants
- since **"x"** is **negative** in the second quadrant, all of the trig ratios that include "x" will be negative
- the only **positive trig ratios** in quadrant II are $\sin\theta$

## Trig Ratios in Quadrant III:




QIII

$$\sin\theta = \frac{opp}{hyp} = \frac{-y}{r}$$

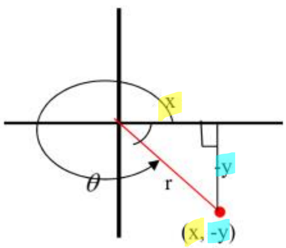
$$\cos\theta = \frac{adj}{hyp} = \frac{-x}{r}$$

$$\tan\theta = \frac{opp}{adj} = \frac{-y}{-x} = \frac{y}{x}$$ } pos

Note:

- "r" is **positive** in all quadrants
- since **"x"** and **"y"** are **negative** in the third quadrant, all but 2 of the trig ratios will be negative
- the only **positive trig ratios** in quadrant III are $\tan\theta$

## Trig Ratios in Quadrant IV:




QIV

$$\sin\theta = \frac{opp}{hyp} = \frac{-y}{r}$$

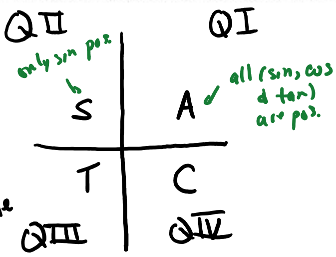
$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$ } pos

$$\tan\theta = \frac{opp}{adj} = \frac{-y}{x}$$

Note:

- "r" is **positive** in all quadrants
- since **"y"** is **negative** in the fourth quadrant, all of the trig ratios that include "y" will be negative
- the only **positive trig ratios** in quadrant IV are $\cos\theta$

Tricks to remember WHICH QUADRANTS have POSITIVE trig ratios:



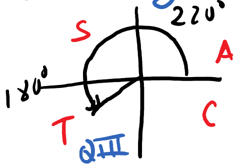

"CAST" Rule

"All Students Take Calculus"

QI

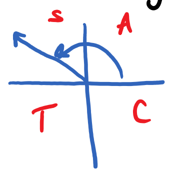
**Example 1**: Without using a calculator, determine whether the following ratios would be positive or negative.

a) sin 220° Neg.

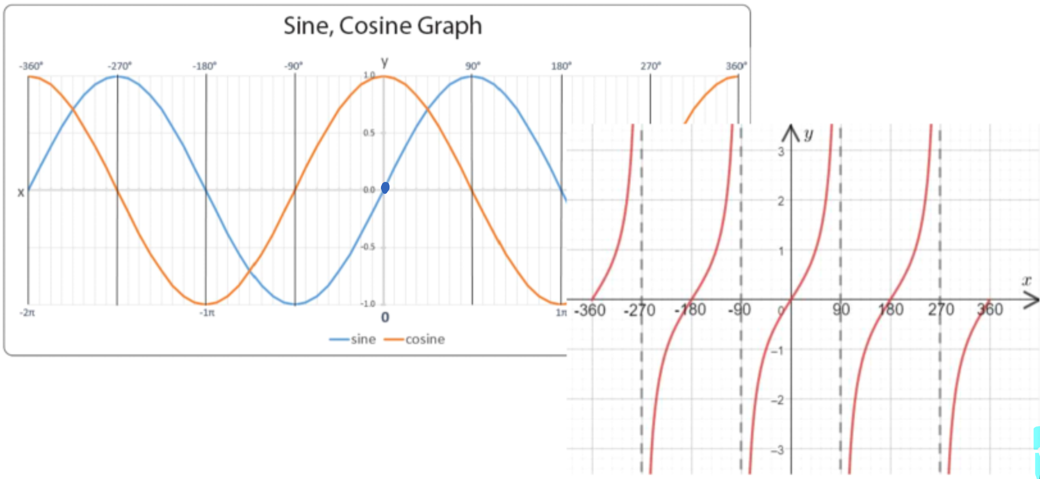



b) cos 330° = pos


c) tan 150° = neg.

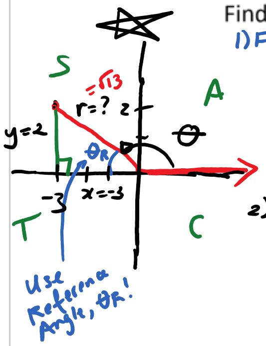



**Example 2**: Study the sin, cos, and tan functions.




$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

**Example 3**: $(-3,2)$ is a point on the terminal arm of the standard position angle $\theta$. Find the exact value of $\sin\theta$, $\cos\theta$, and $\tan\theta$.




1) Find r:

$$\boxed{r^2 = x^2 + y^2}$$

$$= (-3)^2 + 2^2$$

$$r^2 = 13$$

$$r = \sqrt{13}$$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

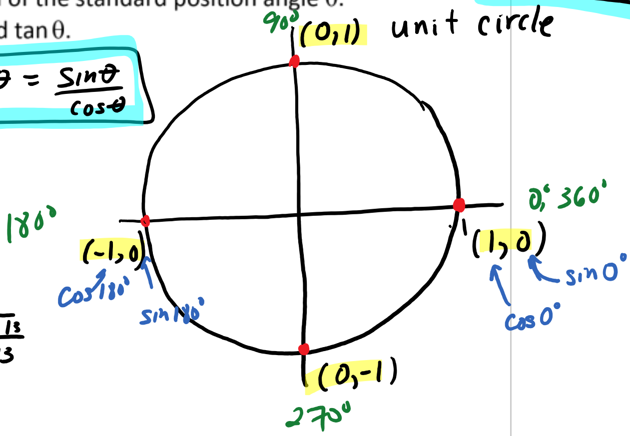
2) SOHCAHTOA

$$\sin\theta = \frac{opp}{hyp} = \frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

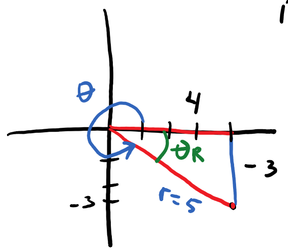
$$\cos\theta = \frac{adj}{hyp} = \frac{-3}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$$

$$\tan\theta = \frac{opp}{adj} = \frac{2}{-3} = -\frac{2}{3}$$

unit circle



**Example 4**: (4, -3) is a point on the terminal arm of the standard position angle θ. Find the exact value of sin θ, cos θ, and tan θ.




1) Find r:

$r^2 = x^2 + y^2$

$= 4^2 + (-3)^2$

$= 16 + 9$

$r^2 = 25$

$r = 5$

2) $\sin\theta = \frac{opp}{hyp} = -\frac{3}{5}$

$\cos\theta = \frac{adj}{hyp} = \frac{4}{5}$

$\tan\theta = \frac{opp}{adj} = \frac{-3}{4}$

**Example 5**: Without using a calculator, find sin 0°, cos 0°, and tan 0°.

**Example 6**: Without using a calculator, find sin 90°, cos 90°, and tan 90°.

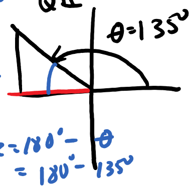
[Answer: 1, 0, undefined]

**Example 7**: Without using a calculator, determine:

① Draw angle

a) sin 135°

QII



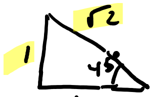
$\theta = 135°$

$\theta_R =$

$\theta_R = 180° - \theta$
$= 180° - 135°$

$\theta_R = 45°$

Find ratio of $\theta_R$



② $\sin\theta_R = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Apply CAST rule!

③

| S | A |
|---|---|
| T | C |

in QII, sin is pos.

so $\sin 135° = \frac{\sqrt{2}}{2}$ ✓

But cos 135°

$\cos\theta_R = \frac{adj}{hyp} = \frac{\sqrt{2}}{2}$

but cos is neg in QII!

so $\cos 135° = -\frac{\sqrt{2}}{2}$

b) tan 300°

**Assignment**: Complete **Trig Snowman** (**to be handed in**!) and **Sec 2.2** p. 96 # 1ac, 2 (use special ratios, not your calculator!), 3, 4abc, 6-8, 16.

Do not print:

solution:

- $x, y, r$ are needed to find the exact sine, cosine, and tangent ratios. Use the Pythagorean Theorem.

$$r^2 = x^2 + y^2$$
$$r^2 = (-3)^2 + (2)^2$$
$$r^2 = 13$$

$r$ is always positive. $r = \sqrt{13}$

- Use the new definitions to find the trigonometric values.

Answer: $\sin\theta = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$, $\cos\theta = -\frac{3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$, $\tan\theta = -\frac{2}{3}$

solution:

- Draw the terminal arm of 0°.
- Choose a point on the terminal arm.
- Determine $x$, $y$, and $r$. $x = 1$, $y = 0$, $r = 1$
- Use the new definitions. $\sin 0° = \frac{0}{1}$, $\cos 0° = \frac{1}{1}$, $\tan 0° = \frac{0}{1}$

Answer: $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$

Example: $\alpha$ is a fourth quadrant angle and $\cos\alpha = \frac{2}{7}$. Find the exact value of $\sin\alpha$.

[Answer: $-\frac{3\sqrt{5}}{7}$]

Example: Without using a calculator, determine:

a) sin 135° b) cos 210° c) tan 300° d) tan 225°