4 Adding and Subtracting Rationals

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ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Adding and subtracting rational expressions uses the same methods as when adding and subtracting fractions; i.e., find a **common denominator**.

Example 1: Evaluate $\frac{8}{15} - \frac{7}{10}$; simplify your answer.

Need common denominator!

① Look at multiples of 15: 15, 30, 45, 60, ...
   Look at multiples of 10: 10, 20, 30, 40, ...

② Lowest Common Denominator (LCD) = 30

③ Multiply each term to get LCD: $\frac{8}{15} \times \frac{2}{2} - \frac{7}{10} \times \frac{3}{3} = \frac{16}{30} - \frac{21}{30} = \frac{-5}{30} = \frac{-1}{6}$

Example 2: Simplify.

$\frac{4}{3x} - \frac{7x}{6}$

$3x \to 3 \cdot x$ $6 \to 2 \cdot 3$ $6x$

① Find LCD = $6x$

② Multiply to get LCD

$\frac{4}{3x} \times \frac{2}{2} - \frac{7x}{6} \times \frac{x}{x}$

$\frac{8}{6x} - \frac{7x^2}{6x}$

$\boxed{\dfrac{8 - 7x^2}{6x}} \quad x \neq 0$

Don't kill kitten! $\dfrac{8 - 7x\cancel{^2}}{\cancel{6x}}$ (crossed out)

Example 3: Simplify

$\frac{1}{3} + \frac{2x-3}{6x^2} - \frac{x-5}{5x}$

3, 6, 9, 12, 15, 18, ..., 30, ..
6, 12, 18, 24, 30, ...
5, 10, 15, 20, 25, 30, ...

LCD: $30x^2$

$\frac{1}{3} \times \frac{10x^2}{10x^2} + \frac{(2x-3)}{6x^2} \times \frac{5}{5} - \frac{1(x-5)}{5x} \times \frac{6x}{6x}$

$\dfrac{10x^2 + 10x - 15 - 6x^2 + -(x-5)(6x)}{30x^2}$

$\dfrac{10x^2 - 6x^2 + 10x + 30x - 15}{30x^2}$

$\boxed{\dfrac{4x^2 + 40x - 15}{30x^2}} \quad x \neq 0$

Example 4: Simplify.

$$\frac{2}{m} - \frac{2m}{m-5}$$

Example 5: Simplify

$$\frac{2x}{x-2} - \frac{x-1}{x+7}$$

Say $\frac{x}{2} - \frac{x}{7}$

$LCD = 14$

$$LCD: (x-2)(x+7)$$

$$\frac{2x}{(x-2)} \cdot \frac{(x+7)}{(x+7)} - \left(\frac{x-1}{x+7}\right)\left(\frac{x-2}{x-2}\right)$$

$$\frac{2x^2 + 14x - [x^2 \overset{-3x}{-2x-x}+2]}{(x-2)(x+7)}$$

$$\frac{2x^2 + 14x - x^2 + 3x - 2}{(x-2)(x+7)}$$

$$\boxed{\frac{x^2 + 17x - 2}{(x-2)(x+7)}}$$ $x \neq 2$
$x \neq -7$

$$\vdots$$

$$\frac{-2m^2 + 2m - 10}{m(m-5)}$$

$$OR \quad \frac{2(-m^2 + m - 5)}{m(m-5)}$$

Example 6: Simplify $\frac{2}{x-4} - \frac{x-8}{x^2-10x+24}$

$$\frac{2}{x-4} - \frac{x-8}{(x-6)(x-4)}$$ $-6 \times 7 = 24$
$-6 + 7 = -10$

$$LCD = (x-4)(x-6)$$

$$\frac{2}{(x-4)} \cdot \frac{(x-6)}{(x-6)} - \frac{x-8}{(x-6)(x-4)}$$

$$\frac{2x - 12 - x + 8}{(x-4)(x-6)} = \frac{x-4}{(x-4)(x-6)} = \frac{1}{x-6} \quad x \neq 6, 4$$

Assignment: "What pet makes the loudest noise?" worksheet, "What happens when the smog lifts in Los Angeles" (do at least 1 from #2-6) worksheet. Optional: Sec 6.3, p. 336 #8, 18

example: Simplify.

$$\frac{t-1}{t^2+5t+4} - \frac{t-2}{t^2+t}$$

exercise: Simplify

$$\frac{2}{x^2+3x+2} - \frac{1}{x^2+x-2}$$

$$\left[\text{Answers: } \frac{-3t+8}{t(t+1)(t+4)} , \frac{x-3}{(x-1)(x+1)(x+2)} \right]$$

solution $\dfrac{t-1}{(t+1)(t+4)} - \dfrac{t-2}{t(t+1)}$

$\text{LCD} = t(t+1)(t+4)$

$\dfrac{t(t-1)}{t(t+1)(t+4)} - \dfrac{(t-2)(t+4)}{t(t+1)(t+4)}$

$\dfrac{t(t-1) - (t-2)(t+4)}{t(t+1)(t+4)}$

$\dfrac{t^2 - t - (t^2 + 2t - 8)}{t(t+1)(t+4)}$

$\dfrac{t^2 - t - t^2 - 2t + 8}{t(t+1)(t+4)}$

$\dfrac{-3t + 8}{t(t+1)(t+4)}$

$\left[\text{Answer:} \quad \dfrac{x-3}{(x-1)(x+1)(x+2)} \right]$

Do Not Print: Additional Examples:

$$\frac{2a-3b}{10a} - \frac{3a-2b}{25b}$$

solution   LCD = $50ab$

$$\frac{(2a-3b)(5b)}{(10a)(5b)} - \frac{(3a-2b)(2a)}{(25b)(2a)}$$

$$\frac{5b(2a-3b)}{50ab} - \frac{2a(3a-2b)}{50ab}$$

$$\frac{5b(2a-3b) - 2a(3a-2b)}{50ab}$$

$$\frac{10ab - 15b^2 - 6a^2 + 4ab}{50ab}$$

$$\frac{-6a^2 + 14ab - 15b^2}{50ab}$$

solution   LCD = $m(m-5)$

$$\frac{2(m-5)}{m(m-5)} - \frac{2m(m)}{m(m-5)}$$

$$\frac{2(m-5) - 2m^2}{m(m-5)}$$

$$\frac{2m - 10 - 2m^2}{m(m-5)}$$

$$\frac{-2m^2 + 2m - 10}{m(m-5)}$$