4 Linear Inequalities in 2 Variables

October 26, 2021 7:47 PM

INEQUALITIES

An inequality is a mathematical statement that compares values that may not be equal.

- $<$ is the symbol for "is less than" $8 < 12$
- $>$ is the symbol for "is greater than" $-8 > -12$
- $\leq$ is the symbol for "is less than or equal to"
- $\geq$ is the symbol for "is greater than or equal to"

Investigate:

Write an inequality, e.g., 3 < 10 or 59 > -16:

$3 < 10$

Multiply both sides by a negative number: Say, -1

$(-1)(3) < (-1)(10)$

$\cancel{-3 < -10}$

$-3 > -10$

Did you need to change anything? If so, what?

If multiply by a negative, change "<" to ">", etc.

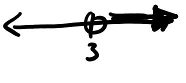
The same rules for equations can be applied to inequalities with one exception!

When multiplying or dividing both sides of an inequality by <u>negative number</u>, the direction of the inequality symbol must be reversed.

To solve any inequality, find <u>all</u> the values of the variable that *satisfies the inequality*.

Example 1: Solve $7 - 2x < 1$ and graph its solution set.

$$\begin{aligned} 7 - 2x &< 1 \\ -7 \quad &\quad -7 \\ \hline -2x &< -6 \\ \frac{-2x}{-2} &\quad \frac{-6}{-2} \\ x &> 3 \end{aligned}$$



$\{x \mid x > 3, x \in \mathbb{R}\}$

- Its graph is on a number line.

Example 2: Solve $5-3x \geq 23$ and graph its solution set.

$$\begin{array}{rcl} 5-3x & \geq & 23 \\ -5 & & -5 \\ \hline -3x & \geq & 18 \\ \div(-3) & & \div(-3) \end{array}$$

$\{x \mid x \leq -6, x \in \mathbb{R}\}$

-6

Example 3: Solve $3x-20 > -2x$ and graph its solution set.

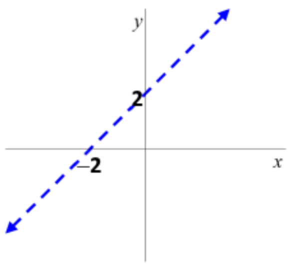
$$\begin{array}{rcl} +2x & & +2x \\ \hline 5x - 20 & > & 0 \\ +20 & & +20 \\ \hline 5x & > & 20 \\ \div 5 & & \div 5 \end{array}$$

$\{x \mid x > 4, x \in \mathbb{R}\}$

4

LINEAR INEQUALITIES IN TWO VARIABLES

To graph the solution of a linear inequality in 2 variables:



- Draw the **boundary line**:
  - Change the inequality to "=" and graph that line.
  - Use a solid line if points on the boundary satisfy the inequality (i.e., ≤ or **≥**).

    Use a **dashed/broken line** if points on the boundary do not satisfy the inequality (i.e., **<** or **>**).

- Solution region: Determine the region with the points that satisfy the inequality.
  - Choose a point on one side of the boundary and check if its coordinates satisfies the inequality. **Trick**: (0,0) is an easy point to test!
  - If the point satisfies the inequality (i.e, is TRUE), shade that region; otherwise, shade the *other* region.

dashed line

slope $= \frac{rise}{run} = \frac{\uparrow}{\rightarrow} = \frac{\Delta y}{\Delta x}$

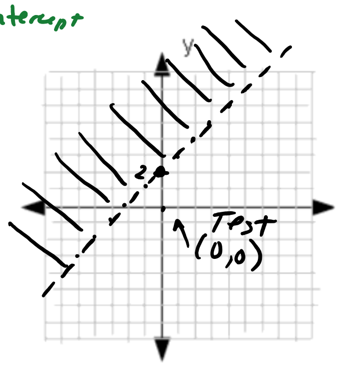
Example 4: Draw the graph of $y > 1x + 2$.

$y = mx + b$ y-intercept

y-int

slope $= m = 1 = \frac{1}{1}$ $\frac{\uparrow}{\rightarrow}$

- Change inequality to '='.
- Graph the boundary line.
- Using the inequality, test a point that's not on the line. Trick: Test (0,0)!
- Solution region: If inequality is TRUE, shade side with the point tested. If FALSE, shade the other side!



$y > x + 2$

$0 > 0 + 2$

$0 > 2$ FALSE

For any inequality statement that is solved for y, the solution will include:

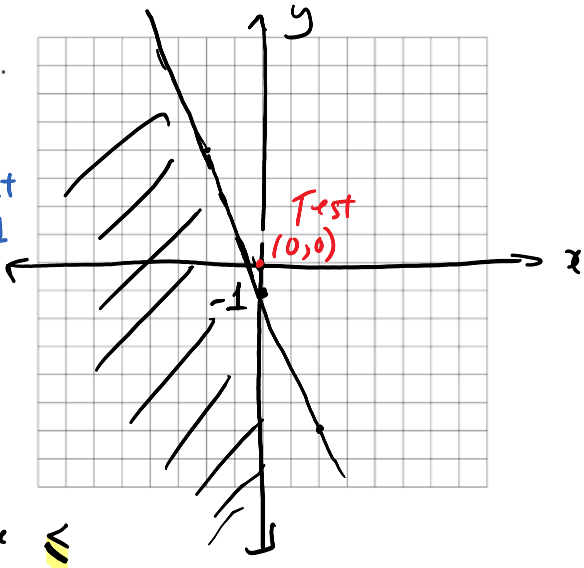
- points **above** the boundary line for **>** **or** $\geq$ inequalities,
- points **below** the boundary line for **<** **or** $\leq$ inequalities,

Example 5: Draw the graph of $y \leq -\frac{5}{2}x - 1$.

Slope, $m = \frac{-5}{2} = \frac{5}{-2}$ y-int $= -1$

$=$ down 5 / right 2

$=$ up 5 / left 2



Use solid line because $\leq$

Test (0,0):

$y \leq -\frac{5}{2}x - 1$

$0 \leq -\frac{5}{2}(0) - 1$

$0 \leq -1$ FALSE! so shade other side!

Example 6: Draw the graph of $2x - 3y < 12$.

$y = mx + b$

$2x - 3y = 12$

$-2x \qquad -2x$

$-3y = -2x + 12$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{12}{-3}$

$y = \frac{2}{3}x - 4$

slope y-int

connect x & y intercepts

Let $x = 0$: $2x - 3y = 12$

$2(0) - 3y = 12$

$\frac{-3y}{-3} = \frac{12}{-3}$

$y = -4$ ↳ $(0, -4)$

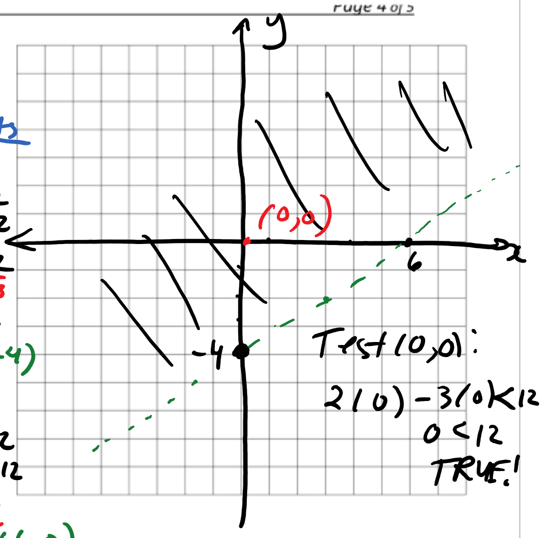
Let $y = 0$:

$2x - 3y = 12$

$2x - 3(0) = 12$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$ ↳ $(6, 0)$



Test (0,0):

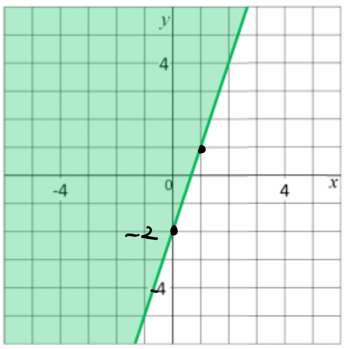
$2(0) - 3(0) < 12$

$0 < 12$

TRUE!

1:45 - 2:00

Example 7: Write the inequality for each graph.



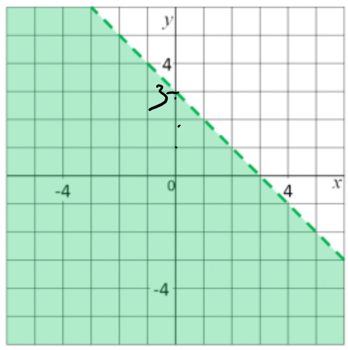
$y = mx + b$

slope, $m = \frac{3}{1} = 3$

$b = -2$ (y-int.)

$\boxed{y \geq 3x - 2}$

SOLID LINE $\geq \ \leq$
Dashed Line $> \ <$
shading above $> \ \geq$
shading below $< \ \leq$



$m = -1$

$b = 3$

Below

dashed

Mr. Slope Guy

$\boxed{y < -x + 3}$

Example 8: Bob started a new workout program. He burns 500 calories per hour jogging and 200 calories per hour lifting weights. He wants to do a combination of these activities and burn at least 2000 calories a week. ~~a)~~ Write the inequality for this situation.
b) Draw the corresponding graph.
c) What are some possible workout combinations that would meet his goal?

a) Define variables

Let j = # hours jogging

Let w = # hours lifting weight

$500j + 200w \geq 2000$ ← at least

SOLID

b) Graph

Let w = 0 to find j-intercept

$500j + 200w = 2000$

$500j + \cancel{200(0)} = 2000$

$\frac{500j}{500} = \frac{2000}{500}$

$j = 4$

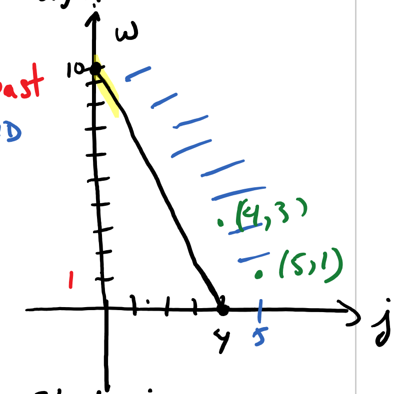
Let j = 0 to find w-intercept

$500j + 200w = 2000$

$\cancel{500(0)} + 200w = 2000$

$\frac{200w}{200} = \frac{2000}{200}$

$w = 10$



Shading:

Test (0,0):

$500j + 200w \geq 2000$

$0 + 0 \geq 2000$

FALSE!

c) possible workouts:

(5, 1) ⇒ 5 hours jogging, 1 hour weights

j w

$500j + 200w \geq 2000$

$500(5) + 200(1) \geq 2000$

$2500 + 200 \geq 2000$

$2700 \geq 2000$ ✓

Pick any point in solution region!

Assignment: Sec 9.1, p. 472 #1-ac, 3-4ace, 8abc (graph by hand), 9, 13, 15