4 Optimization Problems 6.4

May 21, 2019 9:54 PM

FOM 11 **6.4 Optimization Problems I: Creating the Model**

max min

Linear inequalities can be used to solve **optimization problems**, problems in which we find the greatest or least value of functions. The method used to solve such problems is called **linear programming**, and consists of two parts:

1. An **objective function** tells us the quantity we want to maximize or minimize.
2. The system of constraints consists of linear inequalities whose region is called the **feasible solution** with area called the **feasible region**.

Example 1: A company makes motorcycles and bicycles. A restricted work area limits the numbers of vehicles that can be made in one day: no more than 10 motorcycles can be made, no more than 15 bicycles can be made, and no more than 20 vehicles of both kinds can be made. If the profit is \$25 for a motorcycle and \$50 for a bicycle, what should be the daily rate of production of both vehicles to maximize the profits?

Step 1: Define the variables that affect the quantity to be optimized and state any restrictions.

Let x = # of motorcycles

y = # of bikes

$x \in W \rightarrow 0, 1, 2, \ldots$

$y \in W$

Step 2: Identify the quantity that must be optimized.

profit! max

Step 3: Write an objective function.

profit, $P = 25x + 50y$

Step 4: Write a system of linear inequalities to describe all the constraints of the problem and graph the feasible solution.

no more than 10 motocycles:

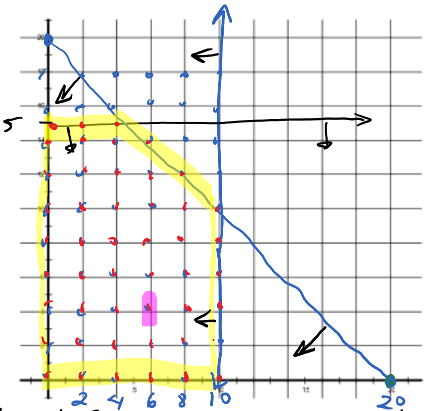
$x \leq 10$

no more than 15 bikes:

$y \leq 15$

no more than 20 all together

$x + y \leq 20$



The region where they intersect is the feasible region

Step 5: Graph each inequality, starting with boundary line:

vertical $x = 10$ $\boxed{\text{HOYVUX}}$

$y = 15$ horizontal

$x + y = 20$

$x = 0$: $0 + y = 20$, $y = 20$

$y = 0$: $x + 0 = 20$, $x = 20$

OR $x + y = 20$
$-x \quad -x$
$y = -x + 20$
$m = -1$ y-int

List a solution:

ex $(6, 4)$
motos bikes

motos bikes

Example 2: Fred is planning an exercise program where he wants to run and swim every week. He doesn't want to spend more than 12 hours a week exercising and he wants to burn at least 1600 calories a week. Running burns 200 calories an hour and swimming burns 400 calories an hour. Running costs \$1 an hour while swimming costs \$2 an hour. How many hours should he spend at each sport to keep his costs at a minimum?

Step 1: Define the variables that affect the quantity to be optimized and state any restrictions.

Let x = # hours runs

y = # hours swims

Step 2: Identify the quantity that must be optimized.

min cost

Step 3: Write an objective function.

$$C = 1x + 2y$$

Step 4: Write a system of linear inequalities to describe all the $\boxed{\text{constraints}}$ of the problem and graph the feasible solution.

no more than 12 hours run & swim:

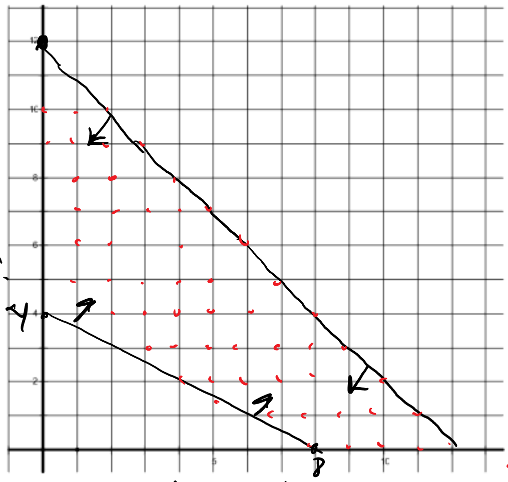
$$x + y \le 12$$

$-x \quad -x$

$$y \le -x + 12$$

• at least 1600 calories

$$200x + 400y \ge 1600$$



$x=0$: $0 + 400y = 1600$
$\frac{400y}{400} = \frac{1600}{400}$
$y = 4$

$y=0$: $\frac{200(x)}{200} + 0 = \frac{1600}{200}$
$x = 8$

Quiz!

Assignment: ~~pg. 330 #1-7~~