4 Projecting Conjectures: Deductive Reasoning, Part 1 (1.4)

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Day 4: Proving conjectures: Deductive reasoning, Part 1 (1.4)

Mathematical proof: a math argument showing that a conjecture is valid (TRUE) or that no counterexample exist

Generalization: a statement that may be True for MOST cases

Deductive Reasoning: a type of reasoning where you make generalizations starting with a general assumption that is known to be valid Ex. All oranges are fruits. All fruits grow on trees. So All oranges grow on trees.

Important "tricks" to use for deductive reasoning proofs!:

A NUMBER:

- Use x for a general number. If they are talking about 2 unrelated numbers, use x and y.
- Write "Let x be ...." to explain your number.

We accept as TRUE

EVEN NUMBER: Any integer multiplied by 2 is an even number.

- This means that $2x$ or 2(any combination of variables and coefficients) will **always** be even.

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ODD NUMBER: If you add 1 to any even integer you will get an odd number.

- This means that $2x+1$ or 2(any combination of variables and coefficients) + 1 will **always** be odd

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CONSECUTIVE NUMBERS: These follow each other in numerical order.

- This means that x, $x+1$, $x+2$, and $x+3$ are 4 numbers that come one after the other numerically.
- Consecutive *even* numbers: $2x$, $2x+2$, $2x+4$, $2x+6$
- Consecutive *odd* numbers: $2x+1$, $2x+3$, $2x+5$, $2x+7$

2, 3, 4

Finishing a Proof:

If proving an answer is...	.. it should look like ....
Even	2(any combination of variable terms)
Odd	2(any combination of variable terms) + 1
Divisible by 3	3(any combination of variable terms)
Divisible by 4	4(any combination of variable terms)
etc.	etc.

Proofs usually involve algebra or transitive property!

If $A=B$
$B=C$
then $A=C$

Example 1:

Choose a number. Using your number:

Ex. 22

1 Multiply by 6
2. Add 14 and divide by 2
3. Add 5 and divide by 3
4. Subtract by your number

- $22 \times 6 = 132$
- $132 + 14 = 146 \rightarrow \frac{146}{2} = 73$
- $\frac{73+5}{3} = \frac{78}{3} = 26$
- $26 - 22 = 4$

Conjecture:

If you follow the above steps, the result is always 4.

Proof:

Let x be any number.

steps:

1) $6x$ multiply by 6

2) $\frac{6x+14}{2} = \frac{\cancel{6x}^{3}}{\cancel{2}} + \frac{14}{2} = 3x + 7$

3) $\frac{3x+7+5}{3} = \frac{3x+12}{3} = \frac{\cancel{3x}}{\cancel{3}} + \frac{12}{3} = x + 4$

4) $x + 4 - x = \boxed{4}$ Always end with 4!

Example 2:

Prove that the sum of two consecutive integers is always odd.

↳ need to end with $2(\ldots\ldots)+1$

Let x = the 1st integer

$x+1$ = the 2nd integer

Prove: sum of x & $x+1$ is odd

$x + x+1$

1st integer 2nd integer

$= 2x + 1$ (odd!)

Example 3:

Prove that the difference (subtract) between consecutive perfect squares is always an odd number.

Let x and $x+1$ be consective #'s.

Prove: $(x+1)^2 - x^2$ will always be odd

$$= (x+1)(x+1) - x^2$$

$$= \cancel{x^2} + x + x + 1 \cancel{- x^2}$$

$$= 2x + 1 \quad \text{odd!}$$

Example 4: What can you deduce?!

All monkeys are mammals. All mammals are vertebrates. Curious George is a monkey. What can be deducted about Curious George?

monkeys = mammals

mammals = vertebrates

Curious G. = monkey

Curious George is a mammal + a vertebrate

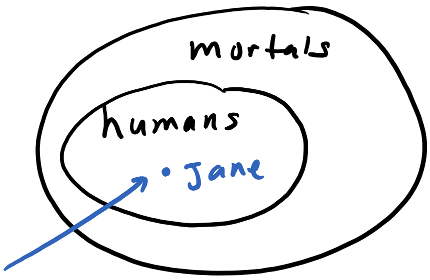
Transitive Property:

If A = B and B = C

then A = C

Example 5: What can you deduce?!

All humans are mortal. Jane is human. What can be deducted about Jane?



human = mortal

Jane = human

$\therefore$ Jane = mortal

Therefore

Example 6: What can you deduce?!
Jamaica (J), Trinidad-Tobago (T), Barbados (Bar) and Bahamas (Bah) are four countries in the Caribbean. All the following statements about their land areas are true. List the countries in order of increasing size.

- Barbados is smaller than Trinidad-Tobago.
- Bahamas is neither the largest nor the smallest.
- At least two countries are larger than Trinidad-Tobago.

Bar, T, Bah, J

Boards: Prove sum of 2 odd #'s is even.

Let: $2x+1$ and $2y+1$ be the odd #'s

$2x+1 + 2y+1$

$= 2x + 2y + 2$

$= 2(x+y+1)$ even

Assignment: Sec. 1.4, p. 31 #2, 4, 5, 7 (number trick), 10 (Optional: 15, 19).