4 Proving Conjectures (1) - Magic Number Tricks + Logic (1.4)

October 13, 2020 8:25 PM

Day 4: Proving conjectures: Deductive reasoning (Part 1) – Magic Number Tricks and Logic (1.4)

Mathematical proof: A math argument showing a conjecture is VALID (TRUE) or no counter example exists

Generalization: a statement that may be TRUE for most cases

Deductive Reasoning: A type of reasoning where generalizations are made from examples (patterns) observed, starting with a general assumption that is known to be TRUE

Important "trick" to use for deductive reasoning proofs!:

A NUMBER:

- Use x for a general number. If they are talking about 2 unrelated numbers, use x and y.
- Write "Let x be a number" to explain your number.

Example 1: Try the following number trick a couple of times. Make a conjecture.

Step	Example	Example
1. Choose a number	0	1
2. Multiply by 6	$(0)(6)=0$	$(1)(6)=6$
3. Add 14 and divide by 2	$\frac{0+14}{2}=7$	$\frac{6+14}{2}=\frac{20}{2}=10$
4. Add 5 and divide by 3	$\frac{7+5}{3}=\frac{12}{3}=4$	$\frac{10+5}{3}=\frac{15}{3}=5$
5. Subtract by your original number	$4-0=4$	$5-1=4$

Conjecture: The end result is always 4.

Now, **prove** the conjecture:

Step	Proof
1. Choose a number	x
2. Multiply by 6	$6x$
3. Add 14 and divide by 2	$\frac{6x+14}{2}=\frac{\cancel{6}^{3}x}{\cancel{2}}+\frac{\cancel{14}^{7}}{\cancel{2}}=3x+7$
4. Add 5 and divide by 3	$\frac{3x+7+5}{3}=\frac{3x+12}{3}=\frac{\cancel{3}x}{\cancel{3}}+\frac{\cancel{12}^{4}}{\cancel{3}}=x+4$
5. Subtract your original number x	$x+4-x$

$=4$

Always end with a 4!

Example 2: Try the following number trick a couple of times. Make a conjecture.

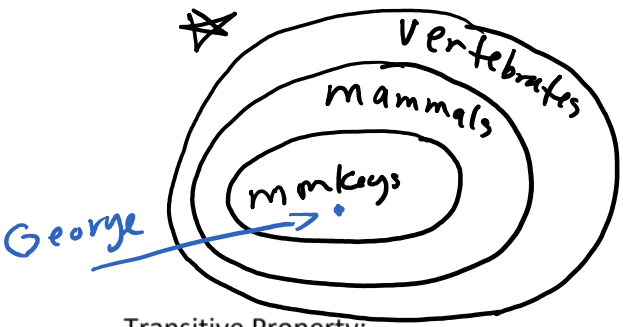
Step	Ex.	Ex.	Proof
1. Choose a number	1	0	x
2. Add 2	$1+2=3$	2	$x+2$
3. Multiply by 3	$3(3)=9$	$2(3)=6$	$3(x+2)=3x+6$
4. Subtract 6	$9-6=3$	$6-6=0$	$3x+6-6=3x$
5. Subtract your original number	$3-1=2$	$0-0=0$	$3x-x=2x$
6. Divide by 2	$2/2=1$	$\frac{0}{2}=0$	$\frac{\cancel{2}x}{\cancel{2}}=x$

Not $3x+2$!

Conjecture: Always end with your original #.

Prove your conjecture using the table above.

Example 3: What can you deduce?!
All monkeys are mammals. All mammals are vertebrates. Curious George is a monkey. What can be deducted about Curious George?



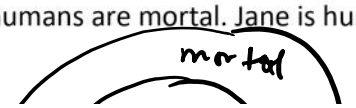
George is a mammal.
and a vertebrate

Transitive Property:

If $A=B$ and $B=C$
then $A=C$

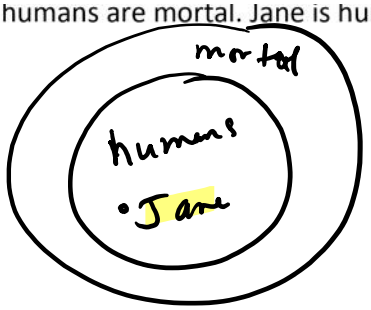
monkeys = mammals
mammals = vertebrate
George = monkey
therefore George = mammal
George = vertebrate

Example 4: What can you deduce?!
All humans are mortal. Jane is human. What can be deducted about Jane?



humans = mortal

All humans are mortal. Jane is human. What can be deducted about Jane?



humans = mortal
Jane = mortal

Jane is mortal

Example 5: What can you deduce?!

Jamaica (J), Trinidad-Tobago (T), Barbados (Bar) and Bahamas (Bah) are four countries in the Caribbean. All the following statements about their land areas are true. List the countries in order of increasing size.

- Barbados is smaller than Trinidad-Tobago.
- Bahamas is neither the largest nor the smallest.
- At least two countries are larger than Trinidad-Tobago.

Barbados < T < Bah < J

Assignment:

- Magic Number Trick: Do Number Tricks Worksheet
- Logic: Do Sec. 1.4, p. 31 #2.