4 Quadratic Inequalities in 2 Variables

November 26, 2019 9:43 PM

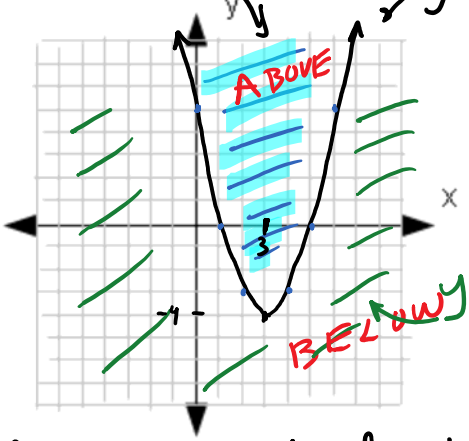
PRE-CALCULUS 11 Chapter 8-9 – Day 6: QUADRATIC INEQUALITIES IN TWO VARIABLES

GRAPHING INEQUALITIES IN TWO VARIABLES

Example 1: Consider the quadratic function $y = x^2 - 6x + 5$

- The **points on the parabola** have coordinates that **satisfy** the function's equation.
- The points **above** the parabola have coordinates that satisfy $y > x^2 - 6x + 5$.
- The points **below** the parabola have coordinates that satisfy $y < x^2 - 6x + 5$.

$y > x^2 - 6x + 5$



Convert to vertex form:

$y = 1x^2 - 6x + 5$ ← y-int

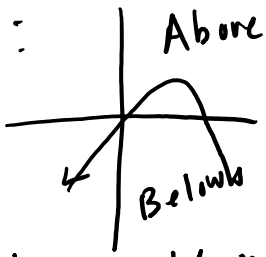
$\frac{b}{2} = \frac{-6}{2} = -3 \rightarrow (-3)^2 = 9$

$= x^2 - 6x + 9 - 9 + 5$

$y = (x-3)^2 - 4$

vertex $(3, -4)$

Note:



→ Always think "above" & "below" instead of "inside" & "outside"!

Same as last time

To graph the solution of a QUADRATIC inequality in 2 variables:
SAME steps are for graphing LINEAR inequalities in 2 variables!

- Change the inequality to "=". Draw the **boundary** line.
  - Draw the line of the equation that corresponds to the inequality.
  - Use a **solid** line if points on the boundary satisfy the inequality (i.e. **≤ or ≥**).
    Use a **dashed/broken line** if points on the boundary do not satisfy the inequality (i.e. **< or >**).
- **Solution region**: Which points satisfy the inequality?
  - Choose a point on one side of the boundary and check if its coordinates satisfies the inequality. **Trick**: An easy point to test is (0,0)!
  - If the coordinates satisfy the inequality (i.e., TRUE), shade that region; *otherwise* shade the *other* region.

Example 2: Draw the graph of $y > x^2 + 4x - 2$ — dashed

convert to vertex form:

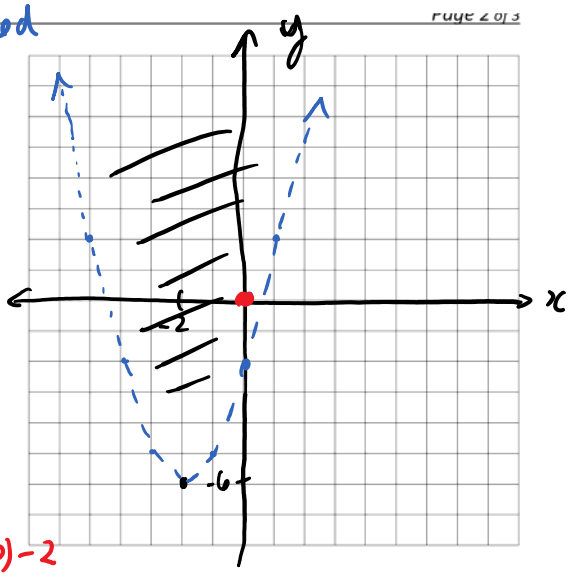
use "equals": $y = x^2 + 4x - 2$

$\frac{b}{2} = \frac{4}{2} = 2 \rightarrow 2^2 = 4$

$y = x^2 + 4x + 4 - 4 - 2$

$y = (x+2)^2 - 6$

vertex $(-2, -6)$



Because $a = 1$, Up 1, 3, 5, ...

Test $(0,0)$:

$0 > 0^2 + 4(0) - 2$

$0 > -2$

TRUE → shade ABOVE (same region as $(0,0)$!)

Example 3: Draw the graph of $y \geq 5 - 2x^2$ — solid

$y \geq -2x^2 + 5$

Recall: $y = x^2 + 5$

basic parabola translated up 5 units

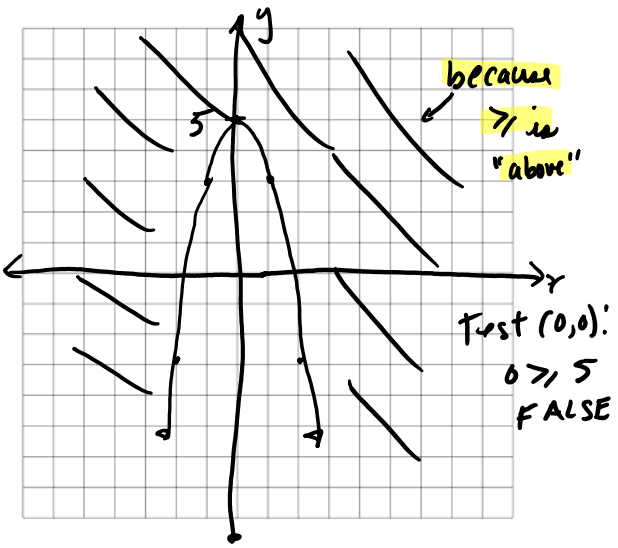
→ $(0,5)$ is vertex.

$a = -2$ so move down

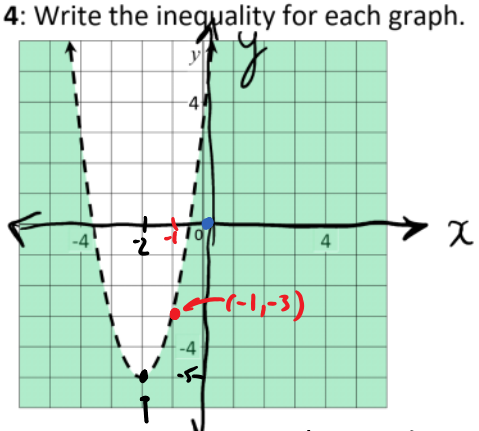
$1 \times 2 = 2$ units

$3 \times 2 = 6$

$5 \times 2 = 10$



Example 4: Write the inequality for each graph.



Use Vertex $(-2,-5)$ + point $(-1,-3)$

p q x y

$y = a(x-p)^2 + q$

$-3 = a(-1-(-2))^2 - 5$

$-3 = a(-1+2)^2 - 5$

$-3 = a - 5$

$+5 \quad +5$

$a = 2$

$y = 2(x+2)^2 - 5$

OR - notice it moves up 2, 6, ...

$2\times1, 2\times3$

$y < 2(x+2)^2 - 5$

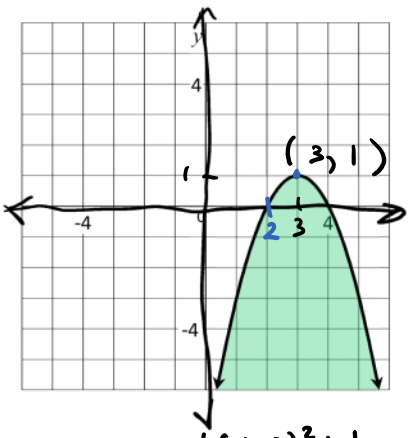
dashed + below

Double check: Test $(0,0)$:

$0 < 2(0+2)^2 - 5$

$0 < 2(4) - 5$

$0 < 3$ TRUE



$y = -1(x-3)^2 + 1$

$y \leq -(x-3)^2 + 1$

{ Below + solid

Assignment: Sec 9.3, p. 496: # 1bc, 3, 4, 6, 8, 9, 13, 16