

4 Radical Equations

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RADICAL EQUATIONS

- A **radical equation** has radical(s) with the variable (e.g., x) in the radicand.
- Ex: $9 - \sqrt{x+5} = 2$
- The solution of any equation is all values of the variable that satisfy the equation.
- Since there is a square root of a variable expression, there may be **restrictions** on the radicand. (You can't **take the square root of a negative number!**)
- **To find the restriction:** 1) Set the radicand to ≥ 0 ; 2) Isolate x to get restriction on x .
- Ex: Given $9 - \sqrt{x+5} = 2$, set radicand $x+5 \geq 0$. The **restriction** is $\therefore x \geq -5$.

SOLVING RADICAL EQUATIONS ALGEBRAICALLY

Radical equations are easier to solve without the radical.

Example 1: Solve $9 - \sqrt{x+5} = 2$

- State **restrictions** on x
- **Isolate** the square root.
- **Square both sides** of the equation.
- Simplify.
- Solve for x .
- **Check root** against restriction.
- **Check root** by plugging into equation.

⑤ Check against restriction:

$$x \geq -5$$

$$44 \geq -5 \quad \checkmark$$

⑥ Check by plugging in:

Plugin $x = 44$

$$9 - \sqrt{x+5} = 2$$

$$9 - \sqrt{44+5} = 2$$

$$9 - \sqrt{49} = 2$$

$$9 - 7 = 2$$

$$2 = 2 \quad \checkmark$$

Ans: $\boxed{x = 44}$

$$x+5 \geq 0$$

$$\begin{array}{r} x+5 \geq 0 \\ -5 \quad -5 \\ \hline x \geq -5 \end{array}$$



Can't take $\sqrt{\quad}$ of a negative!

$$x+5 \geq 0$$

$$\begin{array}{r} x+5 \geq 0 \\ -5 \quad -5 \\ \hline x \geq -5 \end{array}$$

$$9 - \sqrt{x+5} = 2$$

$$\begin{array}{r} 9 \quad \quad \quad -2 \\ -9 \quad \quad \quad -9 \\ \hline -\sqrt{x+5} = -7 \end{array}$$

Mult by -1 on both sides.

$$(-1)(-\sqrt{x+5}) = -7(-1)$$

$$\sqrt{x+5} = 7$$

$$(\sqrt{x+5})^2 = 7^2$$

③ Square both sides

$$\sqrt{(x+5)(x+5)} = 49$$

$$\begin{array}{r} x+5 = 49 \\ -5 \quad -5 \\ \hline x = 44 \end{array}$$

④ Solve for x

EXTRANEIOUS ROOTS

- fails a check!

An **extraneous root** is a value the algebraic steps may produce as roots; however it does not satisfy the original equation and so cannot be a root.

SO ALWAYS CHECK the roots! **REJECT** any extraneous roots!!



Example 2: Solve $\sqrt{x+5} - x = 3$

- ① State **restrictions** on x
- ② Isolate the square root.
- ③ Square both sides of the equation.
- ④ Simplify.
- ⑤ Solve for x .

$$x+5 \geq 0 \quad \text{Can't be neg!}$$

$$x \geq -5$$

$$\sqrt{x+5} - x = 3 \quad \text{② Isolate } \sqrt{}$$

$$\sqrt{x+5} = x+3 \quad \text{③ Square both sides!}$$

$$(\sqrt{x+5})^2 = (x+3)^2 \quad (a+b)^2 = a^2 + 2ab + b^2$$

$$x+5 = x^2 + 6x + 9$$

④ Solve for x

$$0 = x^2 + 5x + 4$$

$$(x+1)(x+4) = 0$$

$$x+1=0 \quad \text{OR} \quad x+4=0$$

$$x=-1 \quad \text{OR} \quad x=-4$$

- ⑤ Check root against restriction.
- ⑥ Check root by plugging into equation.
- ⑦ Check against restriction:

$$x \geq -5$$

$$-1 \geq -5 \quad \checkmark$$

$$-4 \geq -5 \quad \checkmark$$

⑥ Check by plugging in

$$\sqrt{x+5} - x = 3$$

check $x=-1$

$$\sqrt{-1+5} - (-1) = 3$$

$$\sqrt{4} + 1 = 3$$

$$2 + 1 = 3$$

$$3 = 3 \quad \checkmark$$

check $x=-4$

$$\sqrt{x+5} - x = 3$$

$$\sqrt{-4+5} - (-4) = 3$$

$$\sqrt{1} + 4 = 3$$

$$5 = 3 \quad \text{FALSE!}$$

Reject! $x=-4$ is an Extraneous Root!

Ans: $x=-1$

Accept:

Example 3: Solve $\sqrt{2x-1} - x = -2$

Restriction:

$$2x-1 \geq 0$$

$$x \geq \frac{1}{2}$$

$$\begin{array}{r} +x \quad +x \\ \hline \end{array}$$

$$\sqrt{2x-1} = x-2$$

$$(\sqrt{2x-1})^2 = (x-2)^2$$

$$2x-1 = x^2 - 4x + 4$$

$$-2x + 1 \quad -2x + 4$$

$$0 = x^2 - 6x + 5$$

$$\frac{-1}{-1} \times \frac{-5}{-5} = 5$$

$$\frac{-1}{-1} + \frac{-5}{-5} = -6$$

$$(x-1)(x-5) = 0$$

$$x=1 \text{ or } x=5$$

check against restriction:

$$x \geq \frac{1}{2}$$

$$1 \geq \frac{1}{2} \checkmark$$

$$5 \geq \frac{1}{2} \checkmark$$

check
 $x=1$

check $x=5$

$$\sqrt{2(5)-1} - 5 = -2$$

$$\sqrt{10-1} - 5 = -2$$

$$\sqrt{9} - 5 = -2$$

$$3 - 5 = -2$$

$$-2 = -2 \text{ TRUE}$$

accept: $x=5$

check by plugging in

$$\sqrt{2x-1} - x = -2$$

$$\sqrt{2(1)-1} - 1 = -2$$

$$\sqrt{1-1} = -2$$

$$0 = -2 \text{ False!}$$

Reject $x=1$ as extraneous
root!

Assignment: Solving Radical Equations Worksheet (circled questions)

Solve

a) $\sqrt{x-x^2} + x - 1 = 0$

b) $19 - 2\sqrt{1-3x} = 11$

c) $2x - \sqrt{5-2x} = -7$