4 Scale Factors & 3D Objects

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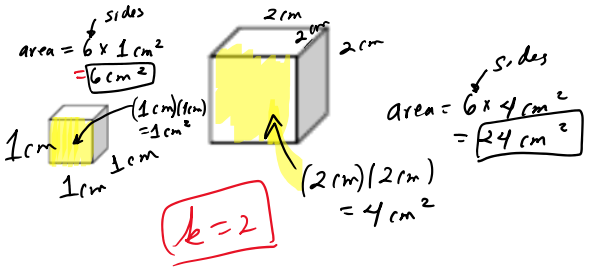
Chapter 8.6: Scale Factors and 3D Objects (Part 1)

What is the relationship between a **linear scale factor** and an **area scale factor**?

length $k = \frac{\text{model length}}{\text{actual length}}$ $k^2 = \frac{\text{model area}}{\text{actual area}}$

What is **surface area**?

Compare the surface areas of a 1-cm cube and a 2 cm cube.



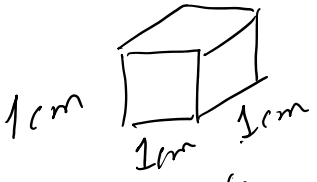
What is the relationship between **scale factor** and **surface area**?

area scale factor $= \frac{\text{model area}}{\text{actual area}} = k^2$

$= \frac{24\ cm^2}{6\ cm^2} = 4 = 2^2 = k^2$

What is **volume**?

Compare the volume of the above ~~cubes~~.

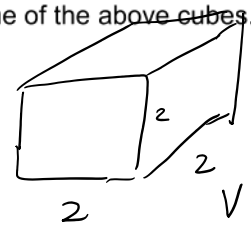


$V = (\text{length})(\text{width})(\text{height})$

$= l \cdot w \cdot h$

$= (1\ cm)(1\ cm)(1\ cm) = 1\ cm^3$

$k = 2$



$V = (2\ cm)(2\ cm)(2\ cm)$

$= 8\ cm^3$

What is the relationship between **scale factor** and **volume**?

volume scale factor $= \frac{\text{model volume}}{\text{actual volume}}$

$= \frac{8}{1} = 8 = 2^3$

$= k^3$

$k^3 = \frac{\text{model volume}}{\text{actual volume}}$

model actual

Example 1: The 1:50 scale model of the dump truck above can carry 1200 cm^3 of sand. How much sand will the actual dump truck be able to carry?

↳ volume = capacity = "space"

$k = \frac{1}{50} = 0.02$

volume scale factor, k^3 = $\frac{\text{model volume}}{\text{actual volume}}$

$(0.02)^3 = \frac{1200\ cm^3}{x}$

$\frac{0.000008x}{0.000008} = \frac{1200}{0.000008}$

Do this 3 times because volume is 3D!

$x = 150\,000\,000\ cm^3 \times \frac{1\ m}{100\ cm} \times \frac{1\ m}{100\ cm} \times \frac{1\ m}{100\ cm}$

$= 150\ m^3$

The actual dump truck can carry 150 m^3 of sand.

Ex: A cylindrical soup can is 10 cm wide and 10 cm tall. The company wants to make a similar shaped can that will hold twice as much soup. What will the diameter and height of the new can be?

By: Bao

↳ double the volume!

∴ volume scale factor $= k^3 = 2$

But what is k?

Take cube root!

Do: $\sqrt[x]{y}$ 2 $\sqrt[x]{y}$ 3

OR shift $\sqrt[3]{\square}$ 2 $= 1.2599\ldots$

$= \boxed{1.26} = k$

$k = \frac{\text{model length}}{\text{actual length}}$

$\frac{1.26}{1} = \frac{x}{10\text{ cm}}$

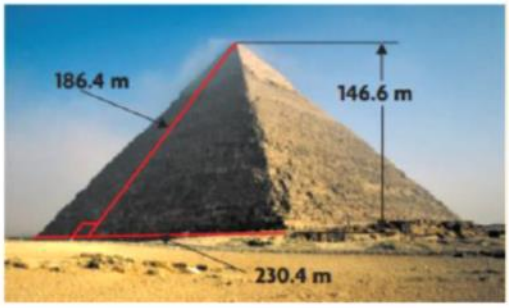
$x = (1.26)(10) = 12.6$ cm

The new can's height = 12.6 cm

width = 12.6 cm

Chapter 8.6: Scale Factors and 3D Objects (Part 2)

Example 1: An artist is building a glass replica of the Great Pyramid of Giza for a gallery. If she uses scale factor of 1:60, how much glass will she need to make the sculpture?

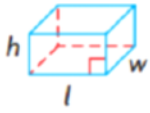
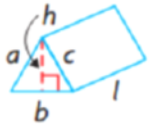
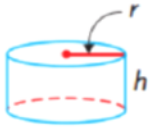
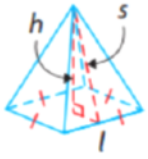
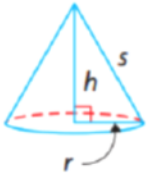
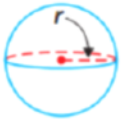


Example 2: Here are two oil spherical oil tanks. The smaller tank has a capacity of 1400 m^3 and the large tank has a capacity of 4725 m^3.

a) If both tanks are filled at the same rate, how many times longer will it take to fill the larger tank than the smaller tank?
b) How many times greater is the radium of the larger tank than the radius of the smaller tank?

Scale Factor	Linear Scale Factor	Area Scale Factor	Volume Scale Factor
Symbol			
Formula			

Formulas We Might Need

Formulas	
Object	**Surface Area and Volume**
rectangular prism 	$SA = 2(lw + lh + wh)$ $V = lwh$
right triangular prism 	$SA = bh + l(a + b + c)$ $V = \frac{1}{2}bhl$
right cylinder 	$SA = 2\pi r^2 + 2\pi rh$ $V = \pi r^2 h$
right pyramid 	$SA = l^2 + 2ls$ $V = \frac{1}{3}l^2h$
right cone 	$SA = \pi r^2 + \pi rs$ $V = \frac{1}{3}\pi r^2 h$
sphere 	$SA = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$