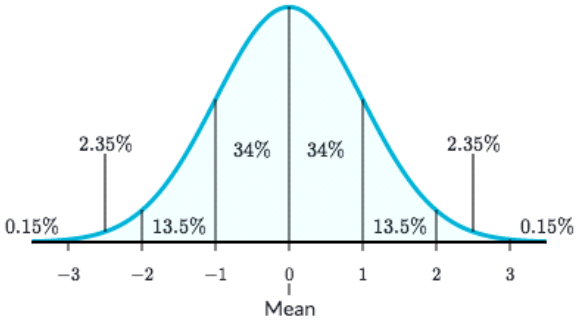
# 5-6 Z-Scores

January 7, 2022 12:06 PM

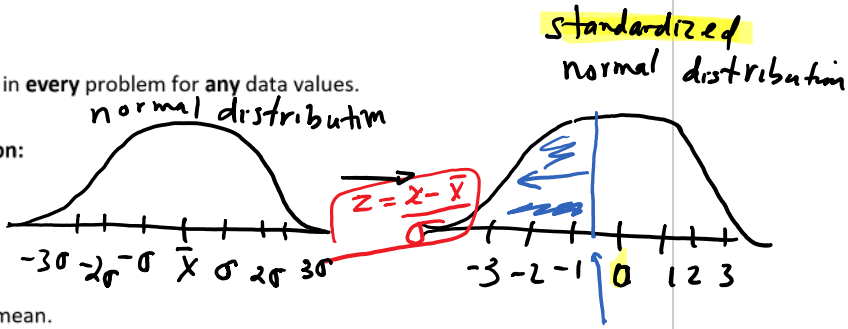



**FOM 11** **5.5 Z-Scores**

Since there are many different possible curves with different values of $\bar{x}$ and σ, we can standardize the curve by transforming **each** score into a **z-score** (a measure of how many standard deviations a value is from the mean).

**Standard Normal Distributions** can be used in **every** problem for **any** data values.

**Properties of a Standard Normal Distribution:**




- Mean is 0.
- Standard Deviation is 1.
- Area under the curve is equal to 1.
- The graph is symmetrical about the mean.
- We use **z** instead of **x** to represent numbers along the horizontal axis.

$$\boxed{z = \frac{(x-\mu)}{\sigma}}$$

data value

$\mu = \bar{x}$ = mean

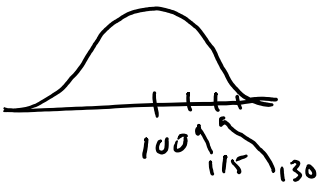
standard deviation

z-score tells what % of data is BELOW your data value, x

- A(z) is the **area under the curve to the left of z**.
- We can find the areas by using a graphing calculator or a z-table.

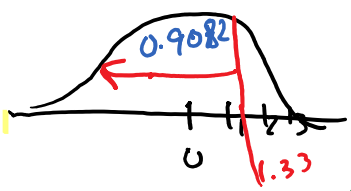
**Example 1:** If IQ scores are normally distributed with a mean of 100 ($\bar{x}$) and standard deviation of 15 (σ) determine:

a. the z-score for 120. (x)




$$z = \frac{x - \bar{x}}{\sigma} = \frac{120 - 100}{15} = \boxed{1.33}$$

b. the probability that a randomly selected person has an IQ less than 120.




(corresponds to IQ of 120)

area A(1.33) = 0.9082

⇒ multiply by 100

⇒ 90.82% of people have IQ < 120

c. the percentage of people with an IQ < 118? $x$

$$z = \frac{x-\bar{x}}{\sigma} = \frac{118-100}{15} = 1.20$$

$A(1.20) = 0.8849 \rightarrow \times 100$

$\Rightarrow$ 88.49% of people have IQ < 118

d. the percentage of people with an IQ < 96? $x$

$$z = \frac{x-\bar{x}}{\sigma} = \frac{96-100}{15} = -0.26$$

$A(-0.26) = 0.3974 \times 100$

$\Rightarrow$ 39.74% have an IQ < 96

$\bar{x} = 2.6$

**Example 2:** The Grade Point Average (GPA) at Burnaby South Secondary is 2.6, with a standard deviation of 0.5. If the top 10% of all students are eligible to attend UBC, what is the minimum GPA needed to attend UBC?

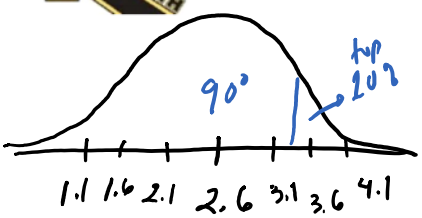
$\sigma = 0.5$

Remember z-scores tables give area to LEFT (or BELOW) the z-score, so Top 10% $\Rightarrow$ bottom 90%

$\Rightarrow$ Find value that gives area closest to 90% $\Rightarrow A(z) = 0.9000$ ?

$\div 100$

Find this (as close as we can!)




$A(1.28) = 0.8997$ — closer to 0.9000 ✓

$A(1.29) = 0.9015$

$$z = \frac{x-\bar{x}}{\sigma}$$

$$\cancel{1.28 = \frac{x-2.6}{0.5}}$$

$x - 2.6 = (1.28)(0.5)$

$x - 2.6 = 0.64$

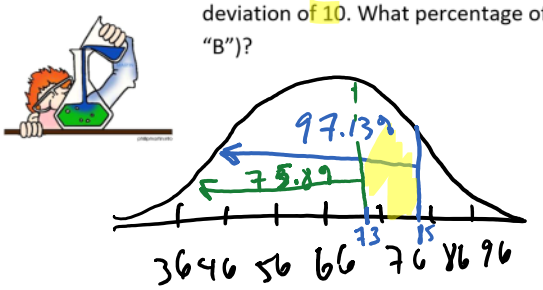
$+2.6 \quad +2.6$

$x = 3.24$

The min. GPA to get into UBC is 3.24.

"Between"

**Example 3:** At Burnaby South Secondary, the average grade for Science is 66, with a standard deviation of 10. What percentage of students get grades between 73 and 85 (i.e., a "B")?




z-score for 73

$$z_{73} = \frac{x-\bar{x}}{\sigma} = \frac{73-66}{10}$$

$z_{73} = 0.70$

$A(0.70) = 0.7580$

$\times 100 \Rightarrow 75.8\%$

get < 73

z-score for 85

$$z_{85} = \frac{85-66}{10}$$

$z_{85} = 1.90$

$A(1.90) = 0.9713$

$\times 100 \Rightarrow 97.13\%$

get < 85

$97.13\% - 75.8\%$

$= 22.3\%$ of students get between 73 & 85.

If 30 students in class, how many?

$$\frac{22.3}{100} \times 30 = 7 \text{ students}$$

**Example 4:** A manufacturer of cell phones has determined a mean of 26 months before a need of repairs, with a standard deviation of 6 months. What length of time should the manufacturer set for this warranty so that less than 10% of all cell phones will need repairs during the warranty period?

① Find z-score for 10% $= 10 \div 100 = 0.1000$

Look for this!

② $A(-1.28) = 0.1003$ is closest!

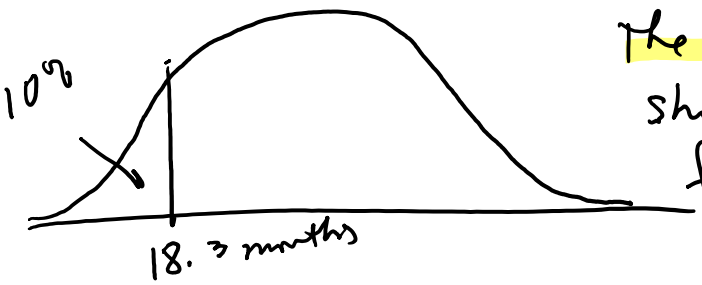
(2) $A(-1.28) = 0.1003$ is closest!
$\uparrow z$

(3) Do algebra to find $x$!

$$z = \frac{x - \bar{x}}{\sigma}$$

$$6(-1.28) = \frac{(x - 26)\cancel{(6)}}{\cancel{6}}$$

$$x - 26 = -7.68$$
$$+26 \quad +26$$
$$x = 18.32 \text{ months}$$




The warranty should be about 18 months for them to fix only 10% of phones.

**Assignment:** Z-scores Worksheet