5-6 Z-Scores

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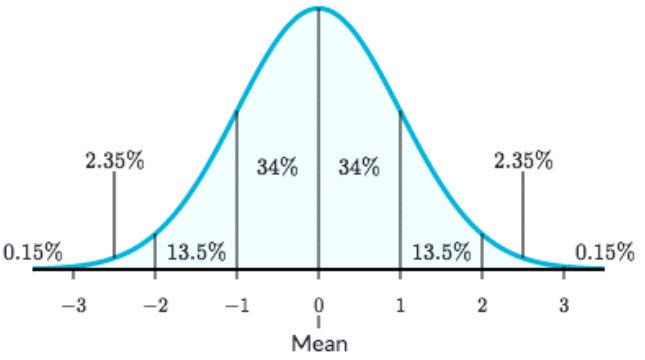


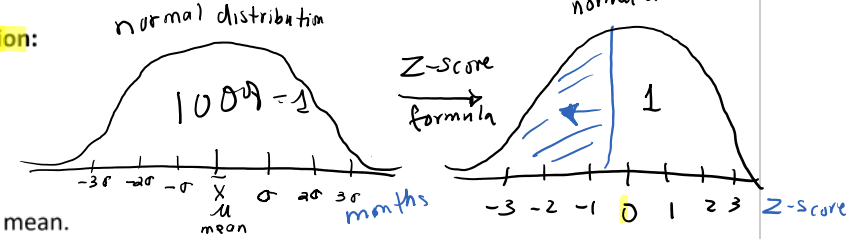
FOM 11 **5.5 Z-Scores**

Since there are many different possible curves with different values of $\bar{x}$ and σ, we can standardize the curve by transforming **each** score into a **z-score** (a measure of how many standard deviations a value is from the mean).

"tweak"

Standard Normal Distributions can be used in **every** problem for **any** data values.

Properties of a Standard Normal Distribution:



- Mean is 0.
- Standard Deviation is 1.
- Area under the curve is equal to 1.
- The graph is symmetrical about the mean.
- We use **z** instead of **x** to represent numbers along the horizontal axis.

z-score

$$z = \frac{(x-\mu)}{\sigma}$$

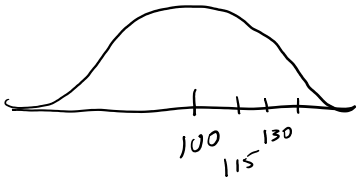
data → x; mean: $\mu, \bar{x}$; standard deviation → σ

Z-score tells what % of the data is BELOW (to the LEFT) of it!

- A(z) is the **area under the curve to the left of z**.
- We can find the areas by using a graphing calculator or a z-table.

Example 1: If IQ scores are normally distributed with a mean of 100 and standard deviation of 15, determine: ($\mu, \bar{x}$; σ)

a. the z-score for 120. (x)



$$z = \frac{x-\mu}{\sigma} = \frac{x-\bar{x}}{\sigma}$$

$$= \frac{120-100}{15}$$

$$= 1.333\ldots$$

$z = 1.33$ ← 2 decimal places

b. the probability that a randomly selected person has an IQ less than 120.

$A(1.33) = 0.9082$ ← from table

$A(1.33)$: area, z-score

x100 Multiply by 100

= 90.82% of people have IQ < 120

have IQ < 120

c. the percentage of people with an IQ < 118?

$z = \frac{x - \bar{x}}{\sigma} = \frac{118 - 100}{15} = 1.20$ need 2 decimals!

$A(1.20) = 0.8849$ ×100

= 88.49% of people have IQ < 118

d. the percentage of people with an IQ < 96?

$z = \frac{x - \bar{x}}{\sigma} = \frac{96 - 100}{15} = -0.266\ldots$

$A(-0.26) = 0.3974$ ×100

= 39.74% of people have IQ < 96

Example 2: The Grade Point Average (GPA) at Burnaby South Secondary is 2.6, with a standard deviation of 0.5. If the top 10% of all students are eligible to attend UBC, what is the minimum GPA needed to attend UBC?

"Top 10%"

"the top 10% cutoff is same as bottom (LEFT) 90%."

⇒ 90 ÷ 100 = 0.9000

Look inside table for 0.9000!

A(1.28) = 0.8997 ← closer!

A(1.29) = 0.9015

so use z = 1.28

Remember z-score gives you the % to the LEFT or BELOW it.

→ Now use algebra to find x

$z = \frac{x - \mu}{\sigma}$

$1.28 = \frac{x - 2.6}{0.5}$ (0.5) Multiply by 0.5 by cross multiplication!

$0.64 = x - 2.6$

+2.6 +2.6

$3.24 = x$

$x = 3.24$

The min GPA needed is 3.24.

Example 3: At Burnaby South Secondary, the average grade for Science is 66, with a standard deviation of 10. What percentage of students get grades between 73 and 85 (i.e., a "B")?

"Between"

Z-score for 73

$z_{73} = \frac{x - \mu}{\sigma} = \frac{73 - 66}{10}$

$z_{73} = 0.70$ need 2 decimal places! (row, column)

Z-score for 85

$z_{85} = \frac{85 - 66}{10} = 1.90$

$z_{85} = 1.90$ (row, column)

A(0.70) = 0.7580 ×100

= 75.8% of students get < 73

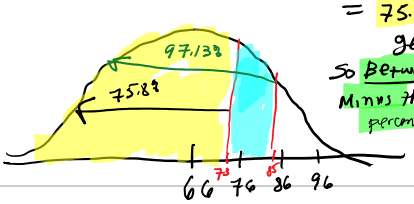
A(1.90) = 0.9713 ×100

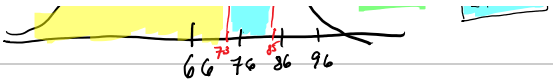
= 97.13% of students get < 85

So Between: 97.13 − 75.80 = 21.33

Minus the percents!

So 21.33% of students get between 73 & 85.

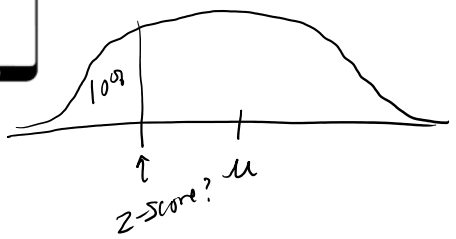




warranty

Example 4: A manufacturer of cell phones has determined a mean of 26 months before a need of repairs, with a standard deviation of 6 months. What length of time should the manufacturer set for this warranty so that less than 10% of all cell phones will need repairs during the warranty period?

↳ BELOW



① Find z-score for 10%:

10 ÷ 100 = 0.1000 Look for this inside the chart!

② Found: A(−1.28) = 0.1003 is closest!

↑ z-score

③ Do algebra!

$$z = \frac{x - \mu}{\sigma}$$

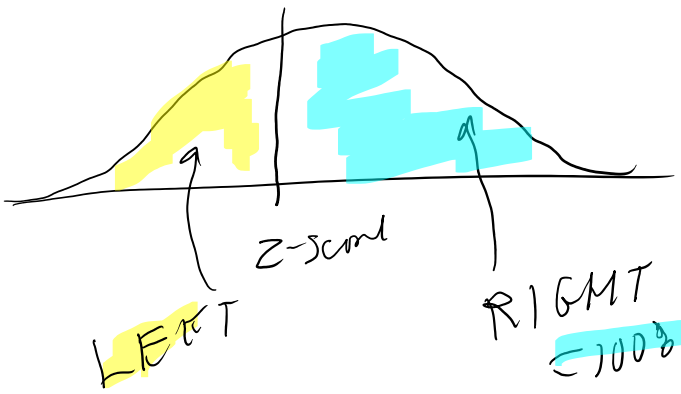
$$(6)(-1.28) = \frac{(x-26)}{6}\,(6)$$

$$-7.68 = x - 26$$

$$+26 \qquad +26$$

$$18.32 = x$$

The warranty should be 18 months for the company to only have to fix 10% of broken phones.



RIGHT = 100% − the percentage to LEFT

Assignment: Z-scores Worksheet