5 Proving Conjectures (2): Proofs with Even/Odd/Consecutive Numbers (1.4)

October 13, 2020 8:26 PM

Day 5: Proving conjectures: Deductive reasoning (Part 2) –Proofs involving Even/Odd/Consecutive Numbers. (1.4)

When we make a conclusion based on statements that we accept as TRUE we are using deductive reasoning. In proofs using deductive reasoning, we use expressions (e.g., for a number, even number, odd number, consecutive numbers, etc.) and algebra.

↳ x ↳ $2x$ $2x+1$

Below are statements that we know are *true*. (Think of them as important "tricks" to use for deductive reasoning proofs!)

A NUMBER:

- Use x for a general number. If they are talking about 2 unrelated numbers, use x and y.
- Write "Let x be ...." to explain your number.

EVEN NUMBER: Any integer multiplied by 2 is an even number.

- This means that $2x$ or 2(any combination of variables and coefficients) will **always** be even. 2

ODD NUMBER: If you add 1 to any even integer you will get an odd number.

- This means that $2x+1$ or 2(any combination of variables and coefficients) + 1 will **always** be odd 3

CONSECUTIVE NUMBERS: These follow each other in numerical order. 2, 3, 4

- This means that x, $x+1$, $x+2$, and $x+3$ are 4 numbers that come one after the other numerically.
- Consecutive *even* numbers: $2x$, $2x+2$, $2x+4$, $2x+6$
- Consecutive *odd* numbers: $2x+1$, $2x+3$, $2x+5$, $2x+7$

Finishing a Proof:

If proving an answer is…	**.. it should look like ….**
Even	2(any combination of variable terms)
Odd	2(any combination of variable terms) +1
Divisible by 3	3(any combination of variable terms)
Divisible by 4	4(any combination of variable terms)
etc.	etc.

☆ Algebra!

add

Example 1: Use deductive reasoning to prove that the sum of an odd number and an even number is always odd. $2(?)+1$

odd + even = odd

Let $2x+1$ be any odd number

Let $2y$ be any even number

* we use x & y because the odd & even # are unrelated

$(2x+1) + 2y$

odd even

$= 2x + 2y + 1$

$= \underbrace{2(x+y)}_{\text{even}} + 1$ odd ✓

Example 2: Prove that the sum of two consecutive integers is always odd.

show $2(\cdots?)+1$

Let *x* = our 1st integer

Let $x+1$ = our 2nd integer

$x + (x+1)$

1st # 2nd #

$= 2x + 1$ odd ✓

Example 3: Prove that the square of an even integer is always even

Let $2x$ be even

$2(\cdots?)$

$(2x)^2$

$= (2x)(2x)$

$= 2(2x \cdot x)$

$= \underbrace{2(2x^2)}_{\text{even}}$ ✓

Example 4:

subtract   exp = 2

Prove that the difference between consecutive perfect squares is always an odd number.

show

$2(\ldots ? \ldots)+1$

Let x be 1^{st} number

Let $x+1$ be 2^{nd} (consecutive) number.

$$(x+1)^2 - x^2$$

$$(x+1)(x+1) - x^2$$

$$\cancel{x^2} + x + x + 1 \cancel{- x^2}$$

$$2x+1 \quad \text{odd} \checkmark$$

~~**Example 5**: The sum of a two-digit number and~~ its reversal is a multiple of 11

Assignment: Deductive Reasoning Worksheet