5 Quadratic Inequalities in 1 Variable

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PRE-CALCULUS 11 Unit 3 – Day 7: QUADRATIC INEQUALITIES IN ONE VARIABLE

- A quadratic equation in one variable: standard form $ax^2 + bx + c = 0$.
- A quadratic **inequality** in one variable will have an inequality symbol (<,≤,>,≥) instead of = .
- A quadratic inequality in one variable can be solved graphically and algebraically.

SOLVING QUADRATIC INEQUALITIES IN ONE VARIABLE GRAPHICALLY

example: Consider the quadratic function $y = x^2 - 6x + 5$ ✓ $y = x^2 - 6x + 5$

1) Graph → vertex form?

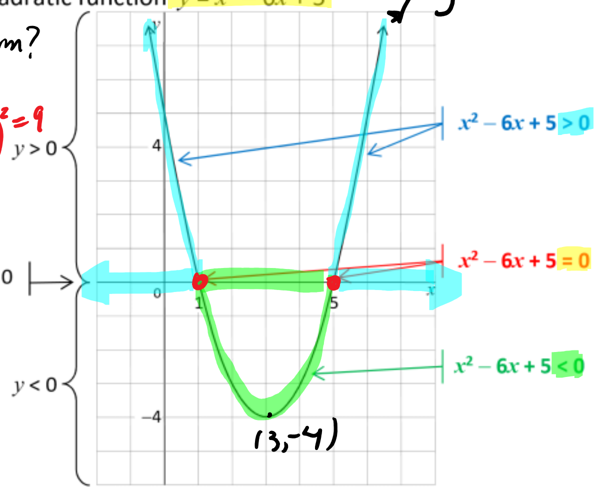
$y = x^2 - 6x + 5$

$b = \frac{-6}{2} = -3 \rightarrow (-3)^2 = 9$

$y = \underbrace{x^2 - 6x + 9}-9+5$

$y = (x-3)^2 - 4$

↳ vertex $(3, -4)$



2) Find roots

↳ x-intercepts

↳ Let $y = 0$ to solve

$x^2 - 6x + 5 = 0$

$(x-1)(x-5) = 0$

$x = 1$ or $x = 5$

3) Solution: · if ≥ or > → look at x-values above y-axis

· if ≤ or < → " " " " below y-axis

· if = → " " where $y = 0$ (x-intercepts)

Solve each of the following:

a) $x^2 - 6x + 5 = 0$

$\{1, 5\}$

b) $x^2 - 6x + 5 > 0$

$\{x \mid x < 1 \text{ or } x > 5, x \in \mathbb{R}\}$

c) $x^2 - 6x + 5 \geq 0$

$\{x \mid x \leq 1 \text{ or } x \geq 5, x \in \mathbb{R}\}$

d) $x^2 - 6x + 5 < 0$

$\{x \mid 1 < x < 5, x \in \mathbb{R}\}$

e) $x^2 - 6x + 5 \leq 0$

$\{x \mid 1 \leq x \leq 5, x \in \mathbb{R}\}$

SOLVING QUADRATIC INEQUALITIES IN ONE VARIABLE ALGEBRAICALLY

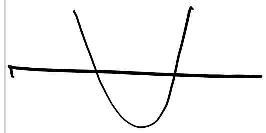
example: Solve the quadratic inequality $x^2 - 6x + 5 < 0$

- **Find the Roots**: Solve $x^2 - 6x + 5 = 0$ algebraically; the **roots** are 1 and 5. Place the roots on a number line; use **closed circles** if these numbers are included in the solution and **open circles** if these numbers do not satisfy the inequality.

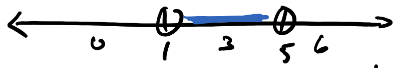
$\leq \; \geq$

$< \; >$

- **Roots and Test Points**: These numbers break up the number line into regions. Test a value from within each region; if it satisfies the inequality, then all the numbers from that region will satisfy it as well.



$x^2 - 6x + 5 < 0$



Test 0: $0^2 - 6(0) + 5 < 0$

$5 < 0$ FALSE $\therefore$ 0 is not in solution region

therefore

Test 3: $3^2 - 6(3) + 5 < 0$

$9 - 18 + 5 < 0$

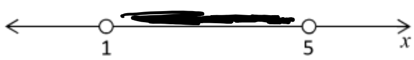
$-4 < 0$ TRUE $\therefore$ 3 is solution region

shade between 1 + 5

Test 6: $6^2 - 6(6) + 5 < 0$

$5 < 0$ FALSE → don't shade

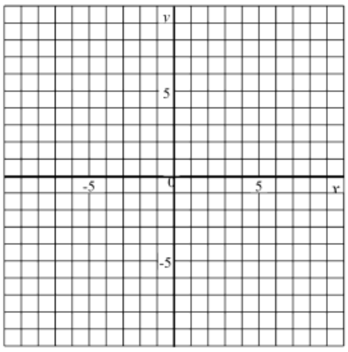
Write the solution set by describing the x-values from all the regions that satisfy the quadratic inequality.



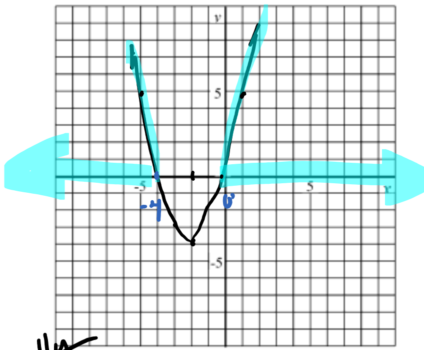
$\{x \mid 1 < x < 5,\ x \in \mathbb{R}\}$

exercise: Solve graphically.

a) $x^2 \le 9$



b) $x^2 + 4x > 0$



graphically

vertex form?

$x^2 + 4x + 0 = 0$

$\frac{b}{2} = \frac{4}{2} = 2 \rightarrow 2^2 = 4$

$y = \underbrace{x^2 + 4x + 4} - 4$

$y = (x+2)^2 - 4$

vertex $(-2, -4)$

$\{x \mid x < -4 \text{ or } x > 0, x \in \mathbb{R}\}$

algebraically

$x^2 + 4x = 0$

$x(x+4) = 0$

$\therefore x = 0$ or $x = -4$

-4 0

Test -5: $(-5)^2 + 4(-5) > 0$

$25 - 20 > 0$

$5 > 0$ TRUE so shade

Test -1: $(-1)^2 + 4(-1) > 0$

$1 - 4 > 0$

$-3 > 0$ FALSE so don't shade

Test 1: $1^2 + 4(1) > 0$

$5 > 0$ TRUE

Assignment: Sec 9.2, p. 486 #1-3, 4, 7-9ac, 13, 15.

exercise: Solve algebraically.

a) $x^2 - 16x + 63 \ge 0$

b) $x^2 + 2x - 1 < 0$

c) $x^2 > 0$

d) $x^2 + 4x + 5 < 0$

CUT OUT:

ORIf the inequality is in factored form, $(x-1)(x-5) < 0$:

use ***Case Analysis***: Test numbers from each region, but just determine if the factors are positive or negative to determine sign of the product.

Test 0:

Test 2:

Test 6:

a) $x^2 - 3x - 54 \geq 0$ Note – this factors very weird when you decompose the middle term! Not a typical factoring type so I replaced the question.

[Answer: $\{x \mid x \leq -6 \text{ or } x \geq 9, x \in \mathrm{R}\}$] b)[Answer: $\{x \mid -1-\sqrt{2} < x < 1-\sqrt{2}, x \in \mathrm{R}\}$]

c) [Answer: $\{x \mid x \neq 0, x \in \mathrm{R}\}$] d) [Answer: ϕ]