5 Quadratic Inequalities in 2 Variables

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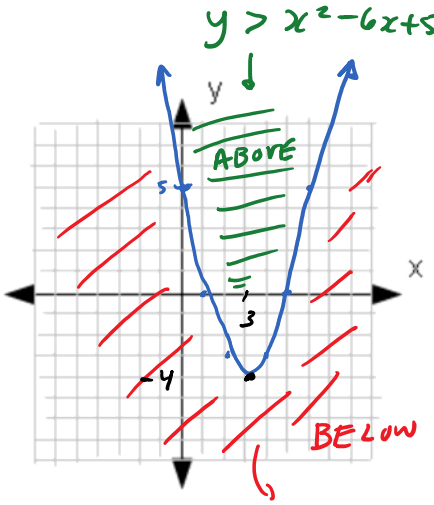
PRE-CALCULUS 11 Chapter 8-9 – Day 5: QUADRATIC INEQUALITIES IN TWO VARIABLES

GRAPHING INEQUALITIES IN TWO VARIABLES

Example 1: Consider the quadratic function $y = x^2 - 6x + 5$

- The **points on the parabola** have coordinates that **satisfy** the function's equation.
- The points above the parabola have coordinates that satisfy $y \geq x^2 - 6x + 5$.
- The points **below** the parabola have coordinates that satisfy $y \leq x^2 - 6x + 5$.



$y = x^2 - 6x + 5$ convert to vertex form

$\frac{b}{2} = \frac{-6}{2} = -3 \rightarrow (-3) = 9$

$y = x^2 - 6x + 9 - 9 + 5$

$y = (x-3) - 4$

vertex: $(3, -4)$

Note:

- ABOVE is not the same as "inside"
- BELOW is not the same as "outside"

To graph the solution of a QUADRATIC inequality in 2 variables:
SAME steps are for graphing LINEAR inequalities in 2 variables!

- Change the inequality to "=". Draw the **boundary** line.
  - Draw the line of the equation that corresponds to the inequality.
  - Use a **solid** line if points on the boundary satisfy the inequality (i.e. **≤ or ≥**).
    Use a **dashed/broken line** if points on the boundary do not satisfy the inequality (i.e. **< or >**).
- **Solution region**: Which points satisfy the inequality?
  - Choose a point on one side of the boundary and check if its coordinates satisfies the inequality. **Trick**: An easy point to test is (0,0)!
  - If the coordinates satisfy the inequality (i.e., TRUE), shade that region; *otherwise* shade the *other* region.

Example 2: Draw the graph of $y > x^2 + 4x - 2$ dashed

Convert to vertex form ("=")

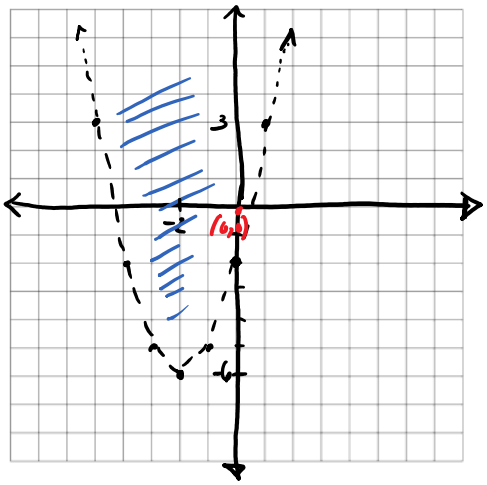
$y = x^2 + 4x - 2$

$\frac{b}{2} = \frac{4}{2} = 2 \rightarrow 2^2 = 4$

$y = x^2 + 4x + 4 - 4 - 2$

$y = (x+2)^2 - 6$

vertex: $(-2, -6)$



Test (0,0): $y > x^2 + 4x - 2$

$0 > 0^2 + 4(0) - 2$

$0 > -2$

TRUE! ∴ shade region with (0,0)

Example 3: Draw the graph of $y \geq 5 - 2x^2$ solid line!

$y \geq -2x^2 + 5$

$y = -2x^2 + 5$

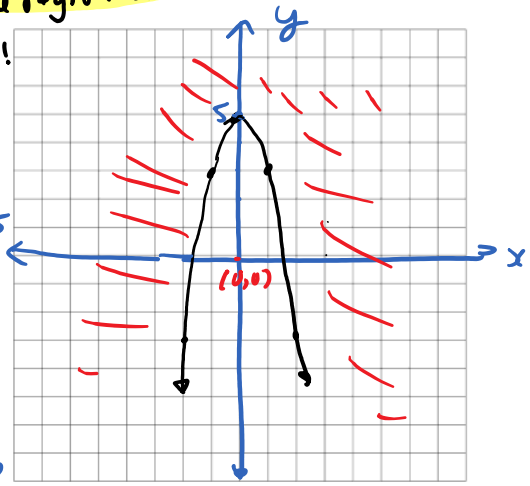
$y = -2(x+0)^2 + 5$

Vertex: (0,5)

$a = -2$: over 1 but down $-2 \times 1 = 2$

$-2 \times 3 = 6$

$-2 \times 5 \times 10$

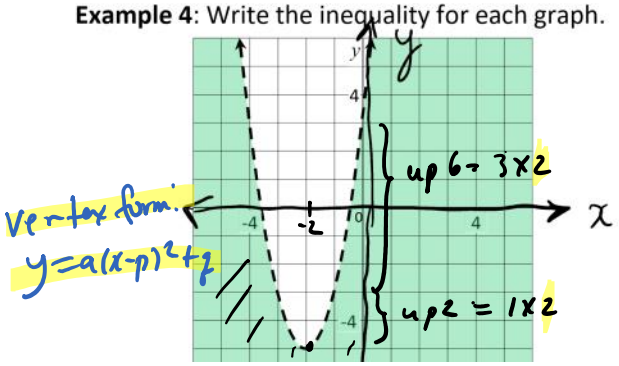


Test (0,0): $y \geq 5 - 2x^2$

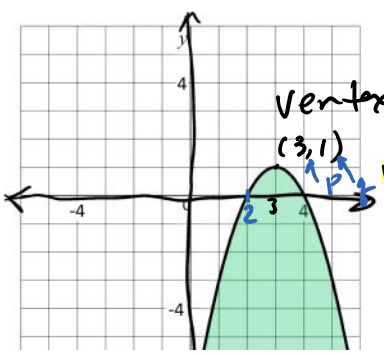
$0 \geq 5 - 2(0)$

$0 \geq 5$ FALSE so shade region without (0,0)

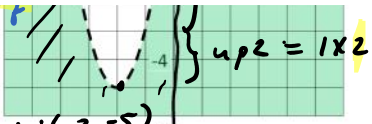
Example 4: Write the inequality for each graph.



Vertex form: $y = a(x-p)^2 + q$



Vertex form: $y = a(x-p)^2 + q$



Vertex: $(-2,-5)$ (p)

We see $a = 2$ (a)

$y = 2(x - -2)^2 - 5$

$y = 2(x+2)^2 - 5$

$y < 2(x+2)^2 - 5$

Dashed Line means $<$ $>$

BELOW means $\leq$ or $<$

another way: $y = a(x-2)^2 - 5$

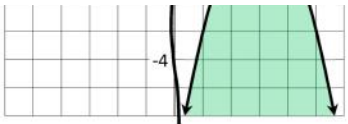
Pick a point, $(x,y) = (-1,-3)$

$-3 = a(-1+2)^2 - 5$

$-3 = a(1)^2 - 5$

$+5 \qquad +5$

$2 = a$



$y \leq -(x-3)^2 + 1$

Same!

Roots are: 2 & 4

$y = -(x-2)(x-4)$

$= -(x^2 - 6x + 8)$

$y \leq -x^2 + 6x - 8$

Assignment: Sec 9.3, p. 496: # 1bc, 3, 4, 6, 8, 9, 13, 16