5 Z-Scores

April 11, 2019 2:53 PM

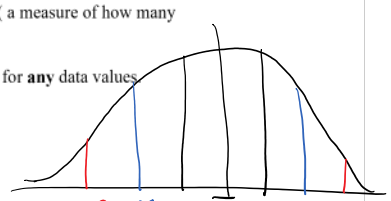
FOM 11 **5.5 Z – Scores** → use when you can't use the 68 – 95 – 99.7 Rule!

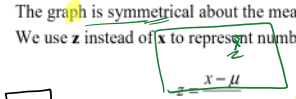
Since there are many different possible curves with different values of μ and σ, we can standardize the curve by transforming **each** score into a **z-score** (a measure of how many standard deviations a value is from the mean).

Standard Normal Distributions can be used in every problem for **any** data values.

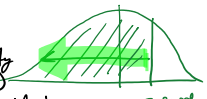
Properties of a Standard Normal Distribution:

- Mean is 0. $\bar{X} = 0$ $\sigma = 1$
- Standard Deviation is 1.
- Area under the curve is equal to 1.
- The graph is symmetrical about the mean.
- We use **z** instead of **x** to represent numbers along the horizontal axis.



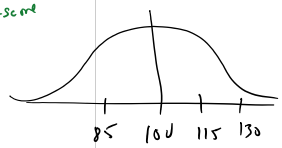


→ tells the probability (or %) of data that are below x



- A(z) is the **area under the curve to the left of z**.
- We can find the areas by using a graphing calculator or a z-table.

Positive z-scores means you are to the RIGHT of mean
negative z-scores → to the LEFT of mean



Example 1: If IQ scores are normally distributed with a mean of 100 and standard deviation of 15, determine:

a. the z-score for 120.

$$z = \frac{x - \bar{x}}{\sigma} = \frac{120 - 100}{15} = 1.33$$

Need 2 decimals! (check 3rd decimal to see if you need to round!)

b. the probability that a randomly selected person has an IQ less than 120.

- Go to positive z-scores table
- Find 1.3 in the z column
- Find 0.03 in top row
- ⇒ 0.9082
- To get percentage, multiply by 100

0.9082 × 100 = 90.82% of population have IQ < 120.

c) what % of people have IQ < 118?

$$z = \frac{x - \bar{x}}{\sigma} = \frac{118 - 100}{15} = 1.20$$

⇒ 0.8849 → ×100 = 88.49%

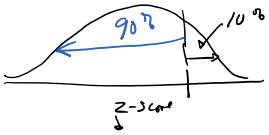
d) what % have IQ < 96?

$$z = \frac{96 - 100}{15} = -0.266$$ (is 5 or more so round up!)

$z = -0.27$

⇒ 0.3936 = 39.36%

Example 2: The GPA at GW Graham Secondary is 2.6, with a standard deviation of 0.5. If the top 10% of all students are eligible to attend UBC, what is the minimum GPA needed to attend UBC?



* z-scores give you area to the LEFT

so 100% − 10% = 90% → = 0.9000

- Look in table → what z-score gives you the closest value to 0.9000
- A(1.28) = 0.8997
  A(1.29) = 0.9015
  which is closer to 0.9000?
  z = 1.28 is closer!

- Find x

$$z = \frac{x - \bar{x}}{\sigma}$$

$$0.5(1.28) = \frac{(x - 2.6)}{0.5} \times 0.5$$

$0.64 = x - 2.6$

$+2.6 \quad +2.6$

$x = 3.24$ is the minimum GPA to get in to UBC.

Example 3: At a high school, the average grade for Science is 66, with a standard deviation of 10. If 20 students with grades between 73 and 85 receive B's, how many students

to get in to UBC.

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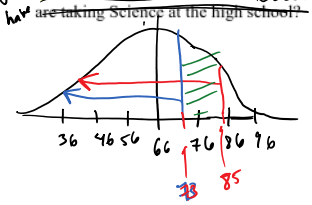
what % of students have ?



z-score for $x = 73$

$z_{73} = \frac{x - \bar{x}}{\sigma}$

$= \frac{73 - 66}{10}$

$z_{73} = 0.7$

z-score for 85

$z_{85} = \frac{85 - 66}{10}$

$z_{85} = 1.9$

for $x = 85$: $A(z_{85}) = A(1.9) = 0.9713 \times 100 = 97.13\%$ of people get less than 85

for $x = 73$: $A(z_{73}) = A(0.7) = 0.7580 \times 100 = 75.80\%$ of people get less than 73

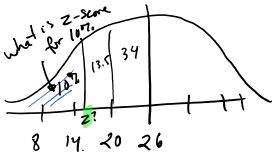
Percent of people who get between 73 & 85 is $A(1.9) - A(0.7)$

$= 97.13 - 75.80$

$= 22.30\%$ get a B on the test.

between 73 & 85

Example 4: A manufacturer of cell phones has determined a mean of 26 months before a need of repairs, with a standard deviation of 6 months. What length of time for this warranty should the manufacturer set so that less than 10% of all cell phones will need repairs during the warranty period?



Find z-score that is closed to $10\% = 10 \div 100 = 0.1000$

$z = -1.28$

because $A(-1.28) = 0.1003$ (closed)

Do algebra to find x:

$z = \frac{x - \bar{x}}{\sigma}$

$6(-1.28) = \frac{(x - 26)}{6} \times 6$

$-7.68 = x - 26$

$+26 \quad +26$

$18.32 = x$

The warranty should be 18 months in order to repair only 10% of the phones.

Assignment: Pg. 264 #5-7(ac), 8, 9, 11, 13-15, 17, 20