6 Discriminant: Predicting the # of Roots

October 11, 2021 11:44 AM

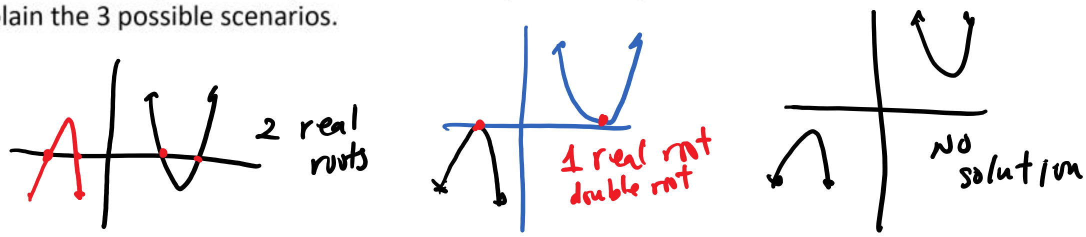
THE DISCRIMINANT

The Quadratic Formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, finds the roots of $ax^2 + bx + c = 0$.

The radicand of a radical is what is inside the radical symbol.

The **discriminant** is the radicand from the Quadratic Formula, $\mathbf{b^2 - 4ac}$.

Warm up review: How many real roots can a quadratic equation have? Draw and explain the 3 possible scenarios.



The value of the discriminant can be used to discriminate between the different types of roots of a quadratic equation **without solving the equation**!

INVESTIGATION: The Mystery of the Discriminant

A. Use the quadratic formula to solve each of the following. Categorize each solution with respect to the **roots** being *real* (if so, are they equal or distinct?) or *imaginary* (that is, no solution, i.e., no real roots).

a) $x^2+8x+12 = 0$

b) $x^2 + 10x + 25 = 0$

c) $x^2 + 2x + 5 = 0$

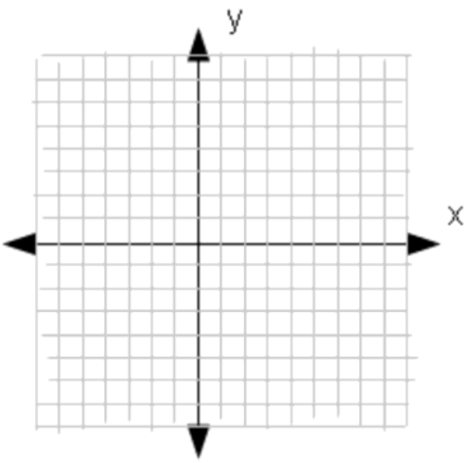
B. Based on your investigation above, what connections did you find between the value of the **discriminant, $b^2 - 4ac$**, and the type of roots? (Discuss the discriminant's value with respect to 0.)

Discriminant: $b^2 - 4ac$ (value with respect to zero)	**Number and type of roots ("<u>nature</u>** of the roots")
a)	
b)	
c)	

C. Graph each of the quadratics by first completing the square to convert the equations from standard form into vertex form. For each, plot the vertex, then sketch the graph and record the roots (i.e., solution/x-intercepts). Note the number of roots.

a) x^2+8x+12 = 0

Convert to vertex form:

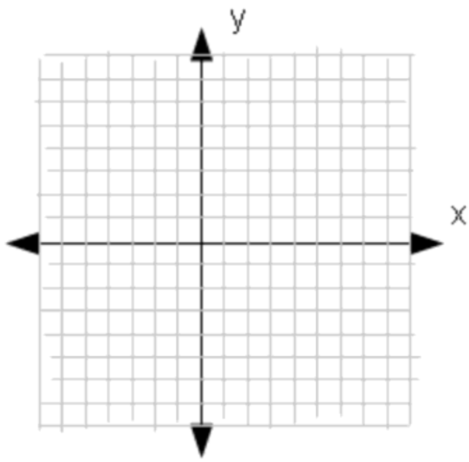


Vertex:

Roots:

b) $x^2 + 10x + 25 = 0$

Convert to vertex form:

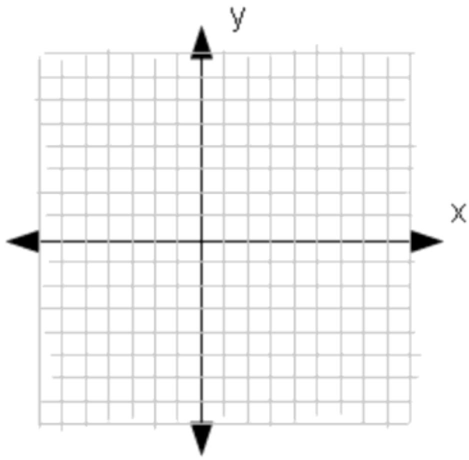


Vertex:

Roots:

c) $x^2 + 2x + 5 = 0$

Convert to vertex form:



Vertex:

Roots:

D. Conclusions: For each case, what links can you make between:

$\sqrt{-1} = i$ (imaginary number)

- The **discriminant** (compare it with respect to 0) and
- The **nature of the roots** (real or imaginary roots, equal or distinct) and
- The **graph** of the equations (use words that include reference to *touching* or *crossing* the x-axis).

$b^2 - 4ac$ (value with respect to zero)	Number and type of roots	Number of x-intercepts
< 0 negative	0 real roots No solution imagmary roots	0
$= 0$	1 real root double root	2 $(x-2)^2 = (x-2)(x-2)$ 1
> 0 positive	2 real roots	2

E. Applying your new knowledge:

Exercise 1: For each of the following, calculate the value of the **discriminant**, $b^2 - 4ac$, and use it to determine i) the nature of the roots, ii) how many x-intercepts does the corresponding graph have?

a) $25x^2 - 20x + 6 = 0$

$a = 25, \ b = -20, \ c = 6$

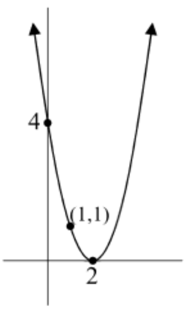
$b^2 - 4ac$
$= (-20)^2 - 4(25)(6)$
$= 400 - 600$
$= -200$
< 0

No soln.
0 real roots

b) $4x^2 - 12x + 9 = 0$

c) $x^2 - x - 5 = 0$

Exercise 2: The graph of $y = ax^2 + bx + c$ is shown. Determine the value of $b^2 - 4ac$.



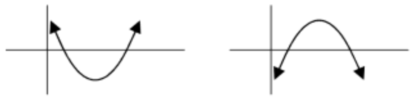
Assignment: Sec 4.4 p. 254 #1-2 (both ade), 8-10, 14, 15. Optional:16, 18. Note – *types of problems*: #8 (area with barn wall), 9 (picture frame/walkway), 10 (subtract 2 numbers), 11 & 13 & 14 (equation given), 12 (surface area), 15 (revenue),16 (right angle triangle).

USING THE DISCRIMINANT TO DETERMINE THE NATURE OF ROOTS

The nature of the two roots from any quadratic equation, $ax^2 + bx + c = 0$, can be:

- not real (i.e., imaginary). This is the case of "no solution".
- equal and real
- distinct and real

Summary

Value of the Discriminant	Nature of the Roots for $ax^2 + bx + c = 0$	Graph and zeros of the associated function $y = ax^2 + bx + c$
$b^2 - 4ac > 0$ (positive discriminant)	2 distinct, real roots	
$b^2 - 4ac = 0$ (zero discriminant)	1 distinct root 2 equal real roots 1 double root	
$b^2 - 4ac < 0$ (negative discriminant)	2 imaginary roots No real roots No solution	

Omit due to time:

Exercise 2: Without graphing, determine how many x-intercepts the graph of $y = x^2 + 2x + 7$ has.

Extension: Suppose you have a quadratic $x^2 - 6x + c = 0$. For what value(s) of c will it have

a) two imaginary roots (i.e., no real solution; no real roots)?

b) distinct real roots?

c) two equal, real roots?