

## 6 Finding Factored Form from Equation given Graph; Word Problems

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FOM 11

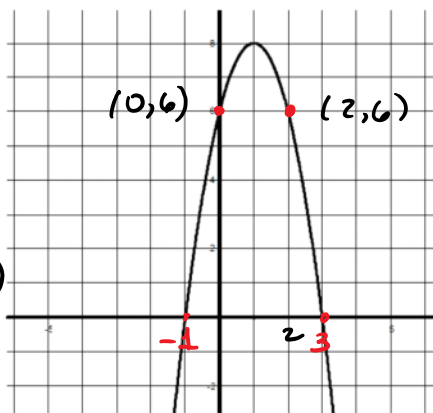
Ch 7 Day 6: Finding Factored Form EQUATION GIVEN GRAPH and Word Problems

### Find the Factored Form Equation of a Quadratic Function Given its Graph

**Example 1:** Determine the EQUATION in FACTORED FORM,  $y = a(x - r)(x - s)$ , of the function that defines each graph. Write each function in standard form.

**Step 1:** Look at the graph and identify the **x-intercepts**. Write these, respectively, as  $r$  and  $s$  in equation. (Note that because of the minus, you end up with the opposite sign!)

*x-intercepts:  $x = -1$   
Take opposites!  $x = 3$   
so  $y = a(x + 1)(x - 3)$*



**Step 2:** Pick a point,  $(x, y)$  from the graph. Plug these into the equation to solve for  $a$ .

*Say you pick  $(x, y) = (2, 6)$*

*Solve for  $a$  by plugging  $x$  &  $y$  that we picked*

$$\begin{aligned} y &= a(x + 1)(x - 3) \\ 6 &= a(2 + 1)(2 - 3) \\ 6 &= a(3)(-1) \\ 6 &= -3a \\ a &= -2 \end{aligned}$$

**Step 3:** Write the equation,  $y = a(x - r)(x - s)$ , but USE the numbers you found for  $a$ ,  $r$ , and  $s$ . LEAVE the  $y$  and  $x$  as variables!!

$$\begin{aligned} y &= a(x + 1)(x - 3) \\ y &= -2(x + 1)(x - 3) \end{aligned}$$

*Keep  $x$  &  $y$  as variables!*

*Not  $6 = -2(2 + 1)(2 - 3)$*

Ch 7 Day 6 - Finding Factored Form Equation Given Graph and Word Problems

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### Find the Equation of a Quadratic Function Given its Graph

**Example 2:** Determine the EQUATION in FACTORED FORM,  $y = a(x - r)(x - s)$ , of the function that defines each graph. Write each function in standard form.

**Step 1:** Look at the graph and identify the **x-intercepts**. Write these.

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### Find the Equation of a Quadratic Function Given its Graph

**Example 2:** Determine the EQUATION in FACTORED FORM,  $y = a(x - r)(x - s)$ , of the function that defines each graph. Write each function in standard form.

**Step 1:** Look at the graph and **identify the x-intercepts**. Write these, respectively, as  $r$  and  $s$  in equation. (Note the minus sign!)

$$x = -1$$

$$x = 2$$

$$y = a(x + 1)(x - 2)$$

**Step 2:** Pick a point,  $(x, y)$  from the graph. Plug these into the equation to solve for  $a$ .

$$(x, y) = (3, 2)$$

$$y = a(x + 1)(x - 2)$$

$$2 = a(3 + 1)(3 - 2)$$

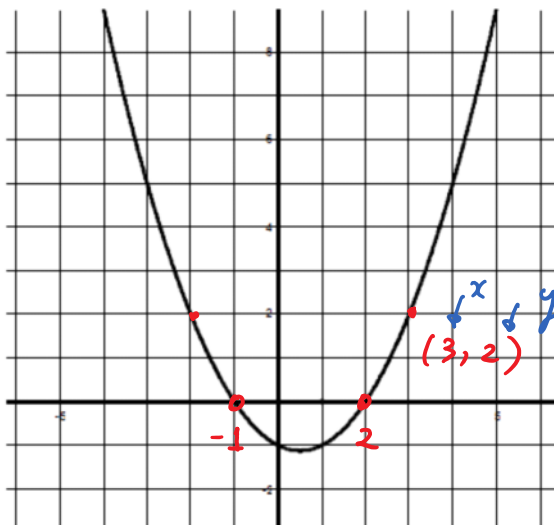
$$2 = a(4)(1)$$

$$\frac{2}{4} = \frac{4}{4}a$$

$$a = \frac{1}{2}$$

**Step 3:** Write the equation,  $y = a(x - r)(x - s)$ , but USE the numbers you found for  $a$ ,  $r$ , and  $s$ . **LEAVE** the  $y$  and  $x$  as variables!!

$$y = \frac{1}{2}(x + 1)(x - 2)$$



**Example 3:** Determine the EQUATION in FACTORED FORM,  $y = a(x - r)(x - s)$ , of the function having x-intercepts at -1 and 4 and a y-intercept of 12.

Take opp sign!

$$y = a(x + 1)(x - 4)$$

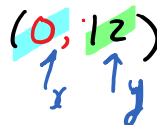
$$12 = a(0 + 1)(0 - 4)$$

$$12 = a(-4)$$

$$-4$$

$$a = -3$$

$$y = -3(x + 1)(x - 4)$$



y-intercept is always at  $x = 0$ !

**Example 4:** Determine the EQUATION in FACTORED FORM,  $y = a(x - r)(x - s)$ , of the function having x-intercepts at -2 and 6 and a y-intercept of -6.

### Solving Word Problems Given the Factored Form Equation of a Quadratic Function

**Example 5:** Burnaby South Secondary students decided to sell baseball caps to raise money for our school. The equation for the profit,  $P$ , they would make based on the  $n$ , the number of units sold is

$$P = -0.5(n - 400)(n - 20)$$



a) What are the break-even points, i.e., how many units would they need to sell **break even** (not lose any money)?

↳ Profit = 0

x-intercepts:  $x = 400$   
 $x = 20$



b) How many units would then need to sell to make a maximum profit?

Take avg!  $\frac{400 + 20}{2} = \frac{420}{2} = 210$  We get max profit if we sell 210 hats.

c) What would their maximum profit be?

Plug in  $x = 210$

$$\begin{aligned} P &= -0.5(x - 400)(x - 20) \\ &= -0.5(210 - 400)(210 - 20) \\ &= -0.5(190)(190) \\ &= \$18,050 \end{aligned}$$

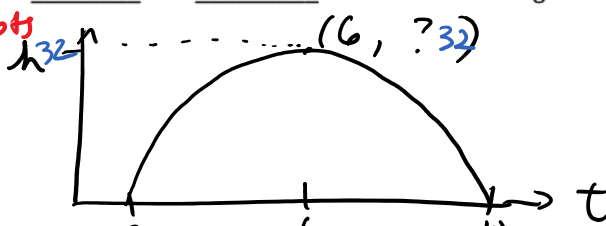
Max profit is \$18,050 if we sell 210 hats.

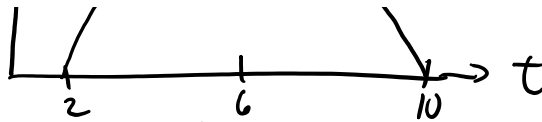
**Example 6:** A water park squirts water into the air. The height can be described by the equation where  $t$  is the time in seconds:

$$h = -2(t - 2)(t - 10)$$

a) What are the **zeros** of this function? 2 and 10. Show this on a rough sketch:

↳ x-intercepts





b) What do the zeros represent?

↳ they represent when water is at ground level (height = 0)

c) What is the axis of symmetry?

Avg!  $t = \frac{2+10}{2} = \frac{12}{2} = 6$

d) Use the axis of symmetry to find the vertex.

plug in  $t=6$ :  $h = -2(t-2)(t-10)$   
 $= -2(6-2)(6-10)$   
 $= 32$   
 vertex  $(6, 32)$

e) What is the maximum height that the water reaches?

the max height is 32 m.

**Example 7:** A soccer ball is kicked into the air. The height can be described by the equation  $x$  is the vertical distance that the ball travels in seconds

$$h = -x(x - 14)$$

a) What are the zeros of this function? \_\_\_\_\_ and \_\_\_\_\_. Show this on a rough sketch:



b) What is the axis of symmetry?

c) Use the axis of symmetry to find the vertex.

d) What is the maximum height that the ball reaches?