6 Linear Programming

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FOM 11   **6.6 Optimization Problems III: Linear Programming**

The solution to an optimization problem is usually found at one of the vertices of the feasible region. To determine the optimal solution to an optimization problem using linear programming, follow these steps:

1. Create an algebraic model that includes:
   - A defining statement of the variables used in you model.
   - Restrictions on the variables. I, W, N, R $<\ >\ \leq\ \geq$
   - A system of linear inequalities that describe the constraints.
   - An objective function that shows how the variables are related to the quantity being optimized. $\hookrightarrow$ max or min
2. Graph the system of inequalities to determine the coordinates of the vertices of the feasible region.   Look at corner points
3. Evaluate the objective function by substituting the values of the coordinates of each vertex.
4. Compare the results and choose the desired solution.
5. Verify your solution(s) satisfies the constraints of the problem situation.

Example 1: A local craft shop produces copper bracelets and necklaces. Each bracelet requires 15 min of cutting time and 10 min of polishing time. Each necklace requires 15 min of cutting time and 20 min of polishing time. There are a maximum of 225 min of cutting time and 200 min of polishing time available each day. The shop makes a profit of $5 on each bracelet and $7 on each necklace sold. How many of each should they make per day to earn the most money?
$\hookrightarrow$ max profit

- Let $x =$ # of bracelets
  $y =$ # of necklaces

- Restriction: $x \in W, y \in W$
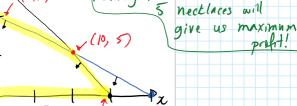 $\hookrightarrow I, W, N, R$? $0,1,2,...$

- Constraints:
  - cutting time: $15x + 15y \leq 225$
  - polishing time: $10x + 20y \leq 200$

- Objective Function: Profit, $P = 5x + 7y$

$15x + 15y = 225$
$x=0$ $0 + 15y = \dfrac{225}{15}$
$\dfrac{15}{15}$
$y = 15$

$y=0$ $15x + 0 = 225$
$x = 15$

$10x + 20y = 200$
$x=0$ $10(0) + 20y = \dfrac{200}{20}$
$\dfrac{20}{20}$
$y = 10$

$y=0$ $\dfrac{10x}{10} = \dfrac{200}{10}$
$x = 20$

Plug corner points into
$P = 5x + 7y$ to find max.   Max!!
$(10,5)$ $P = 5(10) + 7(5) = \$85$
$(0,10)$ $P = 5(0) + 7(10) = \$70$
$(15,0)$ $P = 5(15) + 7(0) = \$75$

Making 10 bracelets and 5 necklaces will give us maximum profit!

Example 2: Fertilizer for a lawn comes in two brands as follows:

	Brand A (kg per bag)	Brand B (kg per bag)
Nitrogen	30	20
Phosphoric acid	2	4
Potash	1	4

A lawn needs at least 120 kg nitrogen, at least 16 kg of phosphoric acid, and at least 12 kg of potash. Brand A costs $22 a bag and Brand B $18 per bag. How many of bags of each brand should be used to minimize the cost? What is the minimum cost?

Assignment: pg. 341 #1, 4, 5, 11-15