

6 Proofs that are Invalid

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FOM 11

Ch1: INDUCTIVE and DEDUCTIVE REASONING Page 15

Day 6: Proofs that are Not Valid (1.5)

Invalid proof: a proof that contains errors in reasoning or invalid assumptions

A proof is invalid if it has/uses:

- A false assumption
- Divides by zero
- Calculation error
- Reasoning error (faulty logic)
- Circular reasoning
- Violates a **premise** (true statement)

Example 1: Is this a valid proof? If not, find the error:

Thomas is a college student. All college students dislike studying.

Therefore, Thomas dislikes studying.

Not a valid proof! FALSE ASSUMPTION!



Example 2: Kimberley claims she can prove that $2 = 3$. Show that she has written an invalid proof.

Her proof:

$$x + y = z$$

$$3x - 2x + 3y - 2y = 3z - 2z$$

$$3x + 3y - 3z = 2x + 2y - 2z$$

$$3(x + y - z) = 2(x + y - z)$$

$$\frac{3(x + y - z)}{x + y - z} = \frac{2(x + y - z)}{x + y - z}$$

$$3 = 2$$

What is wrong?!

Divided by zero!
You can't divide by zero!

$$\begin{array}{r} x + y = z \\ -2 \quad -2 \\ \hline x + y - z = 0 \end{array}$$

sometimes this step is hidden!

Example 3: Is this a valid proof? If not, find the error in reasoning:

Kurt claims to have proven that $2 = 1$. For each statement in his proof, determine if it is valid.

Statement	Reason	Valid?
$x = y$, where $x, y \neq 0$	Given	✓
$x^2 = xy$	Multiply both sides by x	✓
$x^2 - y^2 = xy - y^2$	Subtract y^2 from both sides.	✓
$(x - y)(x + y) = y(x - y)$	Factor out $(x - y)$	✓
$\frac{(x - y)(x + y)}{x - y} = \frac{y(x - y)}{x - y}$ $x + y = y$	Divide both sides by $(x - y)$ but $x = y$ (given) so $x - y = 0$	No! Can't divide by 0!! → invalid proof
$y + y = y$	Substitute y for x since $x = y$	✓
$2y = y$	Simplify	✓
$2 = 1$	Divide both sides by y	✓

Example 4: Is this proof valid? If not, what type of error does it have? Circle the error and correct the proof.

$$2 = 2$$

$$4(2) = 4(1 + 1)$$

$$4(2) + 3 = 4(1 + 1) + 3$$

$$8 + 3 = 6 + 3$$

$$11 = 9$$

Calculation error!
→ invalid proof.

Example 5: Mark claims that $-3 = 3$

Proof: Assume $-3 = 3$

$$(-3)^2 = 3^2$$

$$9 = 9$$

Therefore: $-3 = 3$

Where did Mark go wrong?

Also, circular reasoning

(you can't assume the thing you are trying to prove!)

Example 6: Evan created this number trick: Choose any number. Add 3. Double it. Add 4. Divide by 2. Take away the number you started with.

Each time he tries the trick, he ends up with a 5. His proof does not show this result.

Where did the error occur in his proof?! Find and correct it!

Try:
4
8
12
6
5✓

Let x be any number	x
Add 3	$x + 3$
Double it	$2(x + 3) = 2x + 6$
Add 4	$2x + 6 + 4 = 2x + 10$ ✓
Divide by 2	$\frac{2x + 10}{2} = x + 5$ ✓
Subtract the number you started with	$x + 5 - x = 5$

He did: $2x + 5 - x = x + 5$ (Calculation error!)

Example 7: Prove this number trick by deduction:

1. Pick a 3-digit number with all three digits being the same (ex., 333, 444, etc.)
2. Add the 3 digits together
3. Divide your three-digit number by the sum you found
4. Your answer is 37.

Assignment: Sec 1.5, p. 42# 1 (like Ex. 1 above), 2, 3, 5, 7, 10 (a classic!)