

6 Proofs that are Not Valid (1.5)

January 3, 2020 5:58 PM

Day 6: Proofs that are Not Valid (1.5)

Invalid proof: proof that contains errors in reasoning or invalid assumptions

A proof is invalid if it has/uses:

- Divides by zero
- Calculation error *ex. $3+1=2$*
- Reasoning error
- Circular reasoning \rightarrow when you assume the thing you're to prove is true.
- ~~is~~ based on a false assumption
- Violates a premise (true statement)

Example 1: Is this a valid proof? If not, find the **error in reasoning**:

Thomas is a college student. All college students dislike studying. Therefore, Thomas dislikes studying.

Not valid \downarrow false assumption

Example 2: Kimberley claims she can prove that $2 = 3$. Show that she has written an invalid proof.

Her proof:

$$x + y = z$$

$$(3x - 2x) + (3y - 2y) = (3z - 2z)$$

$$3x + 3y - 3z = (2x) + (2y) - 2z$$
 Rearrange terms

$$3(x + y - z) = 2(x + y - z)$$
 Factor out 3 and 2

$$\frac{3(x + y - z)}{x + y - z} = \frac{2(x + y - z)}{x + y - z}$$
 Divide both sides by $x + y - z$

$$3 = 2$$

What is wrong?!

she said $x + y = z$

but

$$x + y - z = 0$$

Can't divide by 0!
 \Rightarrow invalid proof!

Example 3: Is this a valid proof? If not, find the error in reasoning:

Kurt claims to have proven that $2 = 1$. For each statement in his proof, determine if it is valid.

Statement	Reason	Valid?
$x = y$, where $x, y \neq 0$	Given	✓
$x^2 = xy$	Multiply both sides by x	✓
$x^2 - y^2 = xy - y^2$	Subtract y^2 from both sides	✓
$(x - y)(x + y)$ $= y(x - y)$	Factor out $(x - y)$	✓
$\frac{(x - y)(x + y)}{x - y}$ $= \frac{y(x - y)}{x - y}$ $x + y = y$	Divide both sides by $(x - y)$ but $x = y$ so $x - y = 0$	No! Can't divide by zero!
$y + y = y$	Substitute y for x since $x = y$	✓
$2y = y$	Simplify	✓
$2 = 1$	Divide both sides by y	✓

Example 4: What error, if any, occurs in the following proof?

$$2 = 2$$

$$4(2) = 4(1 + 1)$$

$$4(2) + 3 = 4(1 + 1) + 3$$

$$8 + 3 = 6 + 3 \quad 4(2) = 8 \text{ not } 6!$$

$$11 = 9$$

Calculation
error!

\Rightarrow invalid proof!

Example 5: Mark claims that $-3 = 3$ ← *false assumption*

Proof: Assume $-3 = 3$

$$(-3)^2 = 3^2$$

$$9 = 9$$

Therefore: $-3 = 3$

Where did Mark go wrong?

Also, you can't assume the thing you're trying to prove ⇒ circular reasoning

Example 6: Evan created a number trick in which he always ended with the original number. When he tried to prove the trick, it did not work. Where did the error occur in the proof?!

n	✓	Let n be any number
$n + 4$	✓	Add 4
$2(n + 4) = 2n + 8$	✓	Multiply by 2
$2n + 8$	$2n + 8 + 4 = 2n + 12$	Add 4 <i>calc: error!</i>
$2n + 4$	$\frac{2n + 12}{2} = \frac{2n + 12}{2} = n + 6$	Divide by 2
$n + 1$	$n + 6 - 5 = n + 1$	Subtract 5

Example 7: Prove this number trick by deduction:

1. Pick a 3-digit number with all three digits being the same (ex., 333, 444, etc.)
2. Add the 3 digits together
3. Divide your three-digit number by the sum you found
4. Your answer is 37.

3 digit #: $\underline{x}xx = 100x + 10x + x$

$$x + x + x = 3x$$

$$\frac{100x + 10x + x}{3x} = \frac{x(100 + 10 + 1)}{3x} = \frac{111}{3} = 37$$

valid proof

Assignment: Sec 1.5, p. 42# 1 (like Ex 1 above), 2, 3, 5, 7, 10 (a classic!)