7 Completing the Square (a=1)

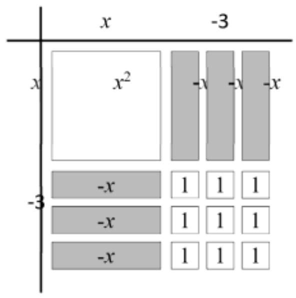
October 5, 2021 12:59 PM

$x - 3$ is a binomial.
$(x - 3)^2$ is the square of the binomial.

When this square of a binomial is expanded, the result is
$x^2 - 6x + 9$ which is a ***perfect square trinomial***.

The algebra tile diagram for the above is shown to the right.

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$



COMPLETING THE SQUARE

Completing the square is an algebraic process where a quadratic polynomial is rewritten so that it contains a ***perfect square trinomial*** so that this trinomial can be *rewritten as the* ***square of a binomial***.

Exercise: Fill in the blank with the term that will produce a perfect square trinomial and then write the trinomial as a square.

Perfect Square Trinomial	Square of a Binomial
$x^2 + 6x + 9$ $a^2 + 2ab + b^2$	a, b: $(x + 3)^2$
$x^2 + 8x + 16$	$(x + 4)^2$
$x^2 - 10x + 25$	$(x - 5)^2$
$x^2 - 12x + 36$	$(x - 6)^2$
$x^2 - 20x + \underline{100}$	$(x - \underline{10})^2$
$x^2 + 16x + \underline{64}$	$(x + \underline{8})^2$
$x^2 + bx + \underline{\frac{b^2}{4}}$ $\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$	$(x + \underline{\frac{b}{2}})^2$
$x^2 - \underline{1}x + \underline{\frac{1}{4}}$ $\left(-\frac{1}{2}\right)^2 = \frac{1}{4}$	$(x - \underline{\frac{1}{2}})^2$
$x^2 + 7x + \underline{12.25}$ $(3.5)^2$	$(x + \underline{3.5})^2$
$ax^2 + bx + c$ $x^2 + \frac{1}{3}x + \underline{\frac{1}{36}}$	$(x + \underline{\frac{1}{6}})^2$

$$\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Recall we can write quadratic functions in different forms:

- Standard form: $ax^2 + bx + c$; $a, b, c \in \mathbb{R}, a \neq 0$ where a is the vertical stretch parameter and c is the y-intercept.
- Vertex form: $y = a(x - p)^2 + q$, where vertex = (p,q) and a is vertical stretch

➢ To convert from *standard form* to *vertex form*, we need to "complete the square" - it is how we get $(x - p)^2$. It also finds q, the **min**imum or **max**imum value of the function, which occurs at $x = p$.

➢ It is straightforward to complete the square when $a = 1$. When $a \neq 1$, WATCH the BRACKETS!

CONVERTING from STANDARD FORM to VERTEX FORM

Example 1: Rewrite $y = \boxed{x^2 - 6x} + 7$ in **vertex form** by **completing the square**. (Note $a = 1$ here).

$y = ax^2 + bx + c$

Steps:

1. Get y alone on 1 side (if not already). ✓
2. What is b? b = -6
3. Divide b by 2. This is the number $(\frac{b}{2})$ that will go in the brackets. -3 $\quad \frac{b}{2} = \frac{-6}{2} = -3$
4. Now take that number and square it. This is the perfect square. $\quad (-3)^2 = 9$ perfect square
5. Copy the first 2 terms given (the $ax^2 + bx$), then add the perfect square, then subtract the perfect square, then write the 3rd term (c). You end up with 5 terms. $\quad y = \underbrace{x^2 - 6x + 9}\underbrace{-9 + 7}_{q}$
6. Rewrite the perfect square trinomial (the 1st 3 terms in the step above) as a square. Add the 4th & 5th terms to form q. $\quad \boxed{y = (x + -3)^2 - 2}$
7. Done! What is the vertex? $(3, -2)$ Min/max value, -2 (q), of the function occurs at x = 3 ? $\quad$ min

Example 2: Rewrite $y = \boxed{x^2 - 8x} + 5$ in vertex form by completing the square.

1. Get y alone on 1 side (if not already).
2. What is b? b = -8
3. Find $\frac{b}{2}$ (This is the number that will go in the brackets.) $\quad \frac{b}{2} = \frac{-8}{2} = -4$
4. Now take that number and square it. This is the perfect square. $\quad (-4)^2 = \boxed{16}$
5. Copy the first 2 terms given (the $ax^2 + bx$), then add the perfect square, then subtract the perfect square, then write the 3rd term (c). You end up with 5 terms. $\quad y = \underbrace{x^2 - 8x + 16}\underbrace{-16 + 5}$
6. Rewrite the perfect square trinomial (the 1st 3 terms in the step above) as a square. Add the 4th & 5th terms to form q. $\quad y = (x + -4)^2 - 11$
7. Vertex? $(4, -11)$ Min/max value? -11 occurs at x = 4?

Assignment: Complete the Square for when a = 1: Sec. 3.3, p. 192 #1, 2, 6a, 7a, 8a.