

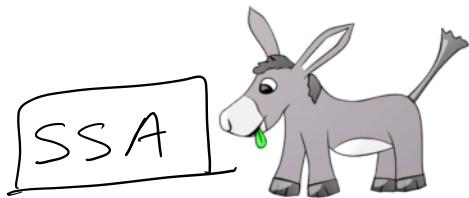
6-7 Sine Law: Ambiguous Case

January 7, 2022 11:26 AM

PRE-CALCULUS 11

Ch 2 – Days 6-7: THE SINE LAW - THE AMBIGUOUS CASE

$$\text{The Law of Sines: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



For SSA, different scenarios may occur:

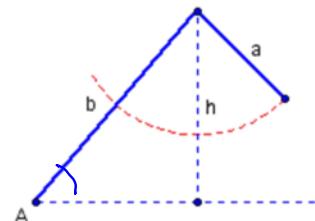
Cases if $\angle A$ is acute:

Find the height: $h = b \sin A$

$a < b$

$a < h$

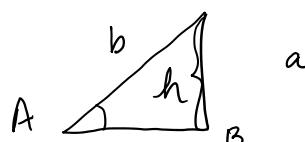
NO triangle



$a < b$

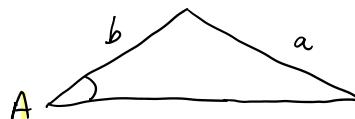
$a = h$

1 right triangle



$a > b$

1 triangle



$a < b$

$a > h$

$h < a < b$

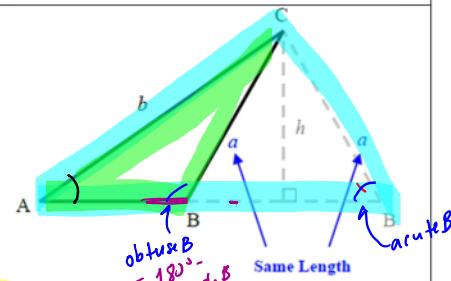
2 triangles

THE AMBIGUOUS CASE WITH THE SINE LAW

Ambiguous means "having more than one possible interpretation".

Apply Sine Rule to find acute version of $\angle B$.

Obtuse version of $\angle B = 180^\circ - \text{acute version of } \angle B$.

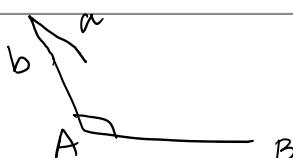


Cases if $\angle A$ is obtuse:

$a < b$ or

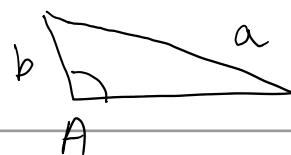
$a = b$

NO triangle



$a > b$

1 triangle

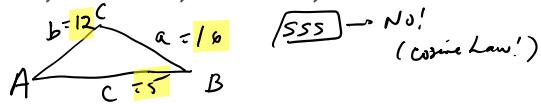


Example 1: For which of these triangles must you consider the ambiguous case?

- a In $\triangle ABC$ $a = 16\text{m}$ $b = 12\text{ m}$ $c = 5\text{ m}$

Example 1: For which of these triangles must you consider the ambiguous case?

- a. In $\triangle ABC$, $a = 16m$, $b = 12m$, $c = 5m$.



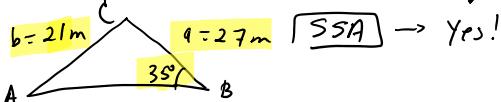
$\boxed{\text{SSS}} \rightarrow \text{No!}$
(cosine law!)

- b. In $\triangle DEF$, $\angle D = 112^\circ$, $e = 110m$, $f = 65m$.



$\boxed{\text{SAS}} \rightarrow \text{No! Not same as SSA!}$

- c. In $\triangle ABC$, $\angle B = 35^\circ$, $a = 27m$, $b = 21m$.



$\boxed{\text{SSA}} \rightarrow \text{Yes!}$

- d. In $\triangle DEF$, $\angle D = 108^\circ$, $\angle E = 52^\circ$, $f = 15m$.



$\boxed{\text{ASA}} \rightarrow \text{no}$

Example 2: Given each SSA situation for $\triangle ABC$, how many triangles are possible?

- a. $\angle A = 30^\circ$, $a = 5$, $b = 10$

First step: find $\text{height} = b \sin A$

$$\begin{aligned} h &= b \sin A \\ &= 10 \sin 30^\circ \end{aligned}$$

$$h = 5$$

$a = h = 5 \Rightarrow \boxed{1 \text{ triangle}}$



- b. $\angle A = 30^\circ$, $a = 7$, $b = 10$

$$h = b \sin A$$

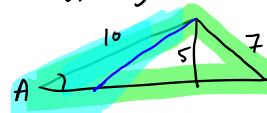
$$= 10 \sin 30^\circ$$

$$h = 5$$

$h < a < b?$

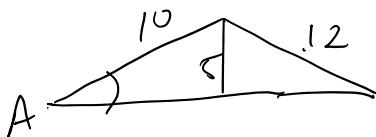
$5 < 7 < 10$ yes!

\hookrightarrow Ambiguous case



- b. $\angle A = 30^\circ$, $a = 12$, $b = 10$

$$\begin{aligned} h &= b \sin A \\ &= 10 \sin 30^\circ \\ &= 5 \end{aligned}$$



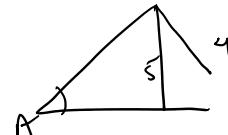
$a > h$
 $12 > 10 \Rightarrow \boxed{1 \text{ triangle}}$

$$h = 5$$

$$a < h$$

$$7 < 5$$

so $\boxed{\text{no triangle}}$



Example 3: Ambiguous Case: $\angle A$ is acute, $a < b, a > h$ 2 triangles:

In $\triangle ABC$, $\angle A = 30^\circ$, $b = 10 \text{ m}$, $a = 6 \text{ m}$. Find $\angle B$.

- **Find height:** $h = b \sin A$
 $= 10 \sin 30^\circ = 5$

- **Use the Sine Law as usual to find acute version of $\angle B$.**

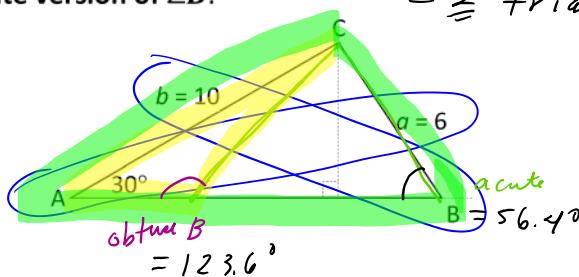
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
 ~~$\frac{\sin B}{10} = \frac{\sin 30^\circ}{6}$~~

$$\frac{6}{\sin B} = \frac{10 \sin 30^\circ}{b}$$

$$= \frac{5}{6}$$

$$B = \sin^{-1}\left(\frac{5}{6}\right)$$

acute $B = 56.4^\circ$



- **Obtuse version of $\angle B = 180^\circ - \text{acute version of } \angle B$:**

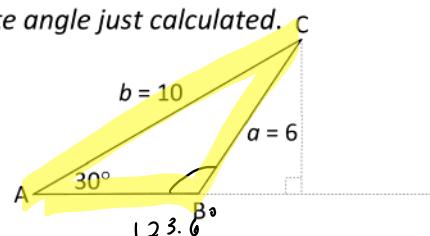
There is also an obtuse angle (2^{nd} quadrant angle) with the **same sine value!**

Its reference angle would be equal to the acute angle just calculated.

$$\text{obtuse } B \approx 180^\circ - 56.4^\circ$$

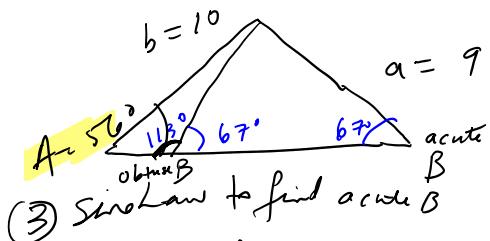
$$B \approx 123.6^\circ$$

There is a second possible triangle for the given information.



Answer:

Example 4: In $\triangle ABC$, $\angle A = 56^\circ$, $b = 10 \text{ m}$, $a = 9 \text{ m}$. Find $\angle B$.



(1) **height:**

$$h = b \sin A$$

$$= 10 \sin 56^\circ$$

$$h = 8.3 \text{ m}$$

(2) **Check:** $h < a < b ?$

$$8.3 < 9 < 10 \rightarrow \text{Yes!} \therefore \text{ambiguous case} \Rightarrow 2 \Delta's$$

(1) **Obtuse B :** $= 180^\circ - \text{acute } B$
 $= 113^\circ$

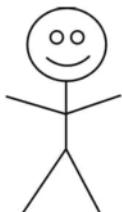
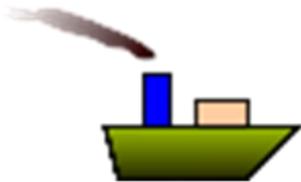
Assignment (Day 6): Ambiguous Case Worksheet #1-3

Example 5: Alex and Bob are holding tethers to a helium balloon that is floating directly above the line through their feet. Alex's tether is 7.8 m long and makes an angle of 36.0° with the ground. Bob's rope is 5.9 m long. What is the distance between Alex and Bob to the nearest tenth of a meter?



Example 6: Sec 2.3, p. 108 #11

A ship from the Canadian Coast Guard has a rotating spotlight that can illuminate up to a distance of 250 m. You are standing at the shore, 500 m from the ship. Your line of sight to the ship makes a 20° angle with the shoreline. What length of shoreline is illuminated by the spotlight?



Assignment (Day 7): Ambiguous Case Worksheet #4-7

Used to assign: Assignment: **worksheet** and **Sec 2.3**, p. 108 # 6, 8, 11, 2 **DO NOT PRINT:**
Cases if $\angle A$ is acute

Case $\angle A$ is acute, $a < b$, $a < h$: no triangle:

In $\triangle ABC$, $\angle A = 30^\circ$, $b = 10$ m, $a = 4$ m. Find $\angle B$.

$$\begin{aligned} h &= b (\sin A) \\ &= 10 (\sin 30^\circ) \end{aligned}$$

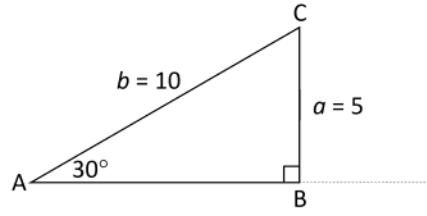
Answer: No triangle exists for the given information.

Case $\angle A$ is acute, $a < b$, $a = h$: one triangle:

In $\triangle ABC$, $\angle A = 30^\circ$, $b = 10$ m, $a = 5$ m. Find $\angle B$.

$$\begin{aligned} h &= b (\sin A) \\ &= 10 (\sin 30^\circ) \end{aligned}$$

$$\begin{aligned} \frac{\sin A}{a} &= \frac{\sin B}{b} \\ \frac{\sin 30^\circ}{5} &= \frac{\sin B}{10} \\ \sin B &= \frac{10 \sin 30^\circ}{5} = 1 \\ B &= 90^\circ \end{aligned}$$



Answer: $\angle B = 90^\circ$; the smallest usable value for a is $b \sin A$.

Case $\angle A$ is acute, $a > b$: one triangle:

Cases if $\angle A$ is obtuse:

Case $\angle A$ is obtuse, $a < b$, $a < h$: no triangle: In $\triangle ABC$, $\angle A = 124^\circ$, $b = 2 \text{ cm}$, $a = 1 \text{ cm}$. Find $\angle B$.

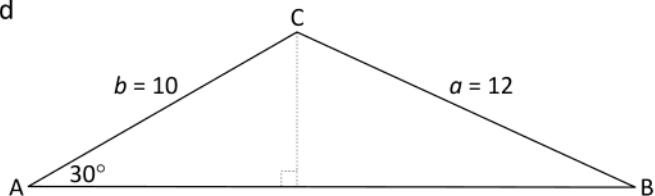
Case $\angle A$ is obtuse, $a > b$: 1 triangle:

In $\triangle ABC$, $\angle A = 145^\circ$, $b = 10 \text{ cm}$, $a = 18 \text{ cm}$. Find $\angle B$.

Do not print:

solution: In $\triangle ABC$, $\angle A = 30^\circ$, $b = 10 \text{ m}$, $a = 12 \text{ m}$. Find $\angle B$.

- Use the Sine Law as usual. $B \approx 24.62432^\circ$
- The obtuse angle calculation results in $B \approx 155.37568^\circ$
The sum of this $\angle B$ and $\angle A$ is larger than 180°
even without $\angle C$.
 \overline{BC} is too long for a second triangle.



Answer: $\angle B \approx 25^\circ$; $a \geq b$.

1. Determine the number of triangles in $\triangle XYZ$, $x = 23.5 \text{ cm}$, $y = 9.8 \text{ cm}$, and $\angle X = 39.7^\circ$.
2. Determine the number of triangles in $\triangle ABC$, $b = 15 \text{ cm}$, $c = 20 \text{ cm}$, and $\angle B = 29^\circ$.