**TRIGONOMETRIC RATIOS OF ANGLES IN STANDARD POSITION**

***y***

***x***

***r***

**P(*x* , *y*)**

θ



Let P(*x*,*y*) represent any point on the terminal arm of the standard position angle θ .



Let *r* represent the distance from the vertex at (0,0) to P(*x*,*y*) ; *r* is always positive because it is a distance.



*x*, *y*, and *r* are the lengths of the sides from the right triangle formed in the diagram.



The Pythagorean Theorem gives the relationship between *x*, *y*, and *r* : ***x*2  +  *y*2  =  *r*2**



The old definitions for the sine, cosine, and tangent of an angle can be applied to angles in standard position using the right triangle in the diagram.



* The ***sine of angle* θ** , **sin θ  =  **



* The ***cosine of angle* θ** , **cos θ  =  **



* The ***tangent of angle* θ** , **tan θ  =  **



With these definitions, trigonometry is no longer limited to acute angles.



Since *x* and *y* can be negative, so can the values of the trigonometric ratios.



**Trig Ratios on the coordinate plane:** Rotating a point around the coordinate plane creates angles in standard position. By dropping a **perpendicular line from the point to the x-axis**, a right triangle is created. Trigonometric ratios occur with respect to the **reference angle.**



(x, y)

x

y

r



Note:

* “r” is **positive** in all quadrants
* since “x” and “y” are also positive in the first quadrant, all of the **trig ratios** in **quadrant I** are **positive**

**Trig Ratios in Quadrant I:**



Note:

* “r” is **positive** in all quadrants
* since **“x”** and **“y”** are **negative** in the third quadrant, all but 2 of the trig ratios will be negative
* the only **positive trig ratios** in quadrant III are  and 

**Trig Ratios in Quadrant III:**

(-x, -y)

-x

-y



r

r



Note:

* “r” is **positive** in all quadrants
* since **“y”** is **negative** in the fourth quadrant, all of the trig ratios that include “y” will be negative
* the only **positive trig ratios** in quadrant IV are  and



**Trig Ratios in Quadrant IV:**

(x, -y)

x

-y





Note:

* “r” is **positive** in all quadrants
* since **“x”** is **negative** in the second quadrant, all of the trig ratios that include “x” will be negative
* the only **positive trig ratios** in quadrant II are  and



**Trig Ratios in Quadrant II:**

(-x, y)

-x

y



r



**Trig Ratios in Quadrant II:**



Note:

* “r” is **positive** in all quadrants
* since **“x”** is **negative** in the second quadrant, all of the trig ratios that include “x” will be negative
* the only **positive trig ratios** in quadrant II are 



(-x, y)

-x

y



r







Note:

* “r” is **positive** in all quadrants
* since **“x”** and **“y”** are **negative** in the third quadrant, all but 2 of the trig ratios will be negative
* the only **positive trig ratios** in quadrant III are 

**Trig Ratios in Quadrant III:**

(-x, -y)

-x

-y



r



r



Note:

* “r” is **positive** in all quadrants
* since **“y”** is **negative** in the fourth quadrant, all of the trig ratios that include “y” will be negative
* the only **positive trig ratios** in quadrant IV are 

**Trig Ratios in Quadrant IV:**

(x, -y)

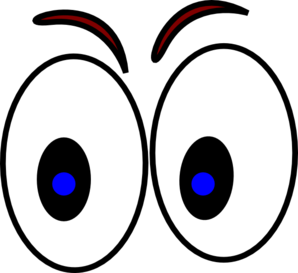
x

-y





Tricks to remember WHICH QUADRANTS have POSITIVE trig ratios:





examples: Without using a calculator, determine whether the following ratios would be positive or negative.



a) sin 220 b) cos 330 c) tan 150



example: (−3,2) is a point on the terminal arm of the standard position angle θ. Find the exact value of sin θ, cos θ, and tan θ.



exercise: α is a fourth quadrant angle and cos α = . Find the exact value of sin α .







example: (4, -3) is a point on the terminal arm of the standard position angle θ. Find the exact value of sin θ, cos θ, and tan θ.



example: Without using a calculator, find sin 0°, cos 0°, and tan 0°.

exercise: Without using a calculator, determine sin 90°, cos 90°, and tan 90°.

[Answer: 1, 0, undefined]

exercise: Without using a calculator, determine:

1. sin 135 b) cos 210 c) tan 300 d) tan 225

HW: Complete Trig Snowman and p. 96 #1-6, 8, 16.Do not print:

solution:

* *x*, *y*, *r* are needed to find the exact sine, cosine, and tangent ratios. Use the Pythagorean Theorem. *r*2   =    *x*2  +  *y*2

*r*2   =  (−3)2 + (2)2

*r*2   =   13

*r* is always positive. *r*    =  

* Use the new definitions to find the trigonometric values.

Answer: sin θ = = , cos θ = = , tan θ = 

solution:

* Draw the terminal arm of 0°.
* Choose a point on the terminal arm.
* Determine *x*, *y*, and *r* . *x* = 1 , *y* = 0 , *r* = 1
* Use the new definitions. sin 0° =  , cos 0° =  , tan 0° = 

Answer: sin 0° = 0 , cos 0° = 1 , tan 0° = 0