**THE LAW OF COSINES**



In every ΔABC,

***The Law of Cosines*: *a*2  =  *b*2  +  *c*2  −  2*bc* cos A**



There are two cases when the Cosine Law can be used, when the given information is:

**Case 1**: two sides and the angle **Case 2**: all 3 sides



between them (“the included angle”)



**Your memory trick to remember:**



**CASE 1: GIVEN TWO SIDES AND THE INCLUDED ANGLE (SAS)**

example: In ΔABC, ∠C = 30°, *a* = 30 cm, and *b* = 50 cm. Find *c* to the nearest hundredth of a centimetre.



*a* = 30

B

C

A

30°

*b* = 50

*c* = **?**



exercise: In ΔJKL, ∠J = 110°, *k* = 3 m, *l* = 4 m. Find *j* to nearest hundredth.



[Answer: 5.76 m]

**CASE 2: GIVEN THREE SIDES (SSS)**

example: In ΔXYZ, *x* = 11.5, *y* = 9.3, and *z* = 16.9. Find the largest angle to the nearest degree.



*y* = 9.3

Z

X

Y

**?**

*x* = 11.5

*z* = 16.9



exercise: In ΔDEF, *d* = 8 m, *e* = 5 m, *f* = 10 m. Find the smallest angle in the triangle to the nearest tenth of a degree.



[Answer: 29.7°]



exercise: Two ships set sail from port P, heading in different directions. The first ship sails 7 km to R and the second ship sails 8 km to Q. If the distance between R and Q is 13 km, what is the angle between the directions of the two ships?



HW: p. 119 #1, 2ac, 3, 4ad, 6, 10, 12, 19, 20

DO NOT PRINT:

*a* = 30

B

C

A

30°

*b* = 50

*c* = **?**

* Consider the oblique triangle ΔABC with side lengths *a*, *b*, and *c*.

Draw an altitude of height *h* from vertex C to point D.

There are now two right triangles, ΔACD and ΔBCD.

* Let *x* represent AD, then *c* − *x* represents BD.

• In ΔACD

A

C

D

*h*

*b*

*x*

cos A =  and

*b* (cos A) =  *x* the Pythagorean Theorem gives

*x* =  *b* cos A  *b*2  =  *h*2  +  *x*2

• In ΔBCD The Pythagorean Theorem gives *a*2  =  *h*2  +  (*c* − *x*)2

*a*2  =  *h*2  +  *c*2  −  2*cx*  +  *x*2

C

D

B

*h*

*a*

*c* − *x*

*a*2  =  *h*2  +  *x*2  +  *c*2  −  2*cx*

*a*2  =       *b*2      +  *c*2  −  2*cx*

*a*2  =  *b*2  +  *c*2  −  2*c*(*b* cos A)

*a*2  =  *b*2  +  *c*2  −  2*bc* cos A

Similarly *b*2  =  *a*2  +  *c*2  −  2*ac* cos B

and *c*2  =  *a*2  +  *b*2  −  2*ab* cos C

solution: In ΔABC, ∠C = 30°, *a* = 30 cm, and *b* = 50 cm. Find *c* to the nearest hundredth of a centimetre.

* Draw a labelled diagram.
* ∠C must be used, so the formula used is

*c*2  =    *a*2   +   *b*2   −   2*ab* cos C

*c*2  =  (30)2  +  (50)2  −  2(30)(50) cos 30°

*c*2  =  900  +  2500  −  3000 cos 30°

*c*2  ≈  801.923 789

*c*   ≈  28.318 259

Answer: *c* ≈  28.32 cm

solution: In ΔXYZ, *x* = 11.5, *y* = 9.3, and *z* = 16.9. Find the largest angle to the nearest degree.

*y* = 9.3

Z

X

Y

**?**

*x* = 11.5

*z* = 16.9

* Draw a labelled diagram.
* The largest angle will be opposite the longest side. *z* is the longest side; ∠Z is the largest angle.
* To find ∠Z in ΔXYZ, the formula used must be

*z*2  =  *x*2  +  *y*2  −  2*xy* cos Z

cos Z  =  

Z  =  

Z  =  

Z  =  cos−1( −0.312 622 72 )

Z  ≈  108.217 359°

Answer: ∠Z ≈  108°