8 Cosine Law

January 7, 2022 11:51 AM

THE LAW OF COSINES

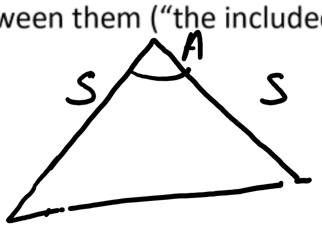
In every ΔABC,

The Law of Cosines:	$a^2 = b^2 + c^2 - 2bc\cos A$
	$b^2 = a^2 + c^2 - 2ac\cos B$
	$c^2 = a^2 + b^2 - 2ab\cos C$

There are two cases when the Cosine Law can be used, when the given information is:

Case 1: two sides and the angle between them ("the included angle")

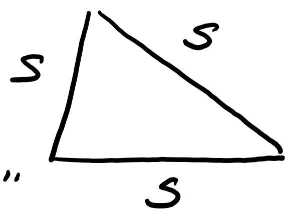
$\boxed{\text{SAS}}$



"SAS sy SSsides!"

Case 2: all 3 sides

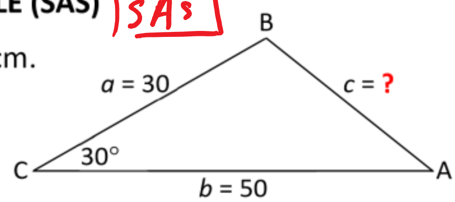
$\boxed{\text{SSS}}$



Your memory trick to remember:

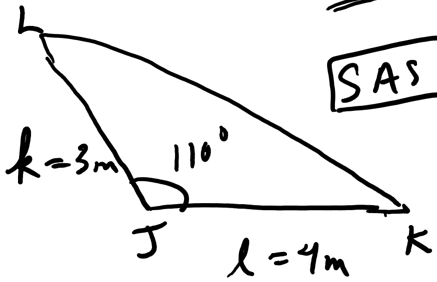
CASE 1: GIVEN TWO SIDES AND THE INCLUDED ANGLE (SAS) $\boxed{\text{SAS}}$

Example 1: In ΔABC, $\angle C = 30°$, $a = 30$ cm, and $b = 50$ cm. Find c to the nearest hundredth of a centimetre.



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 30^2 + 50^2 - 2(30)(50)\cos 30°$$
$$= 900 + 2500 - 3000\cos 30°$$
$$\sqrt{c^2} = \sqrt{801.923}$$
$$\boxed{c = 28.32 \text{ cm}}$$

Example 2: In ΔJKL, ∠J = 110°, *k* = 3 m, *l* = 4 m. Find *j* to nearest hundredth.



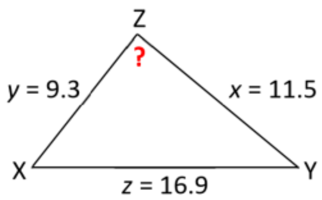
$\boxed{\text{SAS}}$

$$j^2 = k^2 + l^2 - 2kl\cos J$$
$$= 3^2 + 4^2 - 2(3)(4)\cos 110°$$
$$= 9 + 16 - 24\cos 110°$$
$$\sqrt{j^2} = \sqrt{33.2}$$
$$\boxed{j = 5.76\text{ m}}$$

[Answer: 5.76 m]

CASE 2: GIVEN THREE SIDES (SSS)

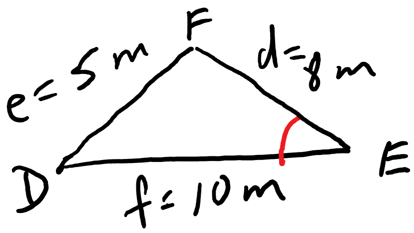
Example 3: In ΔXYZ, *x* = 11.5, *y* = 9.3, and *z* = 16.9. Find the largest angle to the nearest degree.



$$z^2 = x^2 + y^2 - 2xy\cos Z$$
$$(16.9)^2 = \underbrace{(11.5)^2 + (9.3)^2}\ - 2(11.5)(9.3)\cos Z$$
$$285.61 = 218.74 - 213.9\,\boxed{\cos Z}$$
$$-218.74 \qquad -218.74$$
$$\frac{66.87}{-213.9} = \frac{\cancel{-213.9}\,\boxed{\cos Z}}{\cancel{-213.9}}$$
$$\cos Z = -0.3126$$
$$Z = \cos^{-1}(-0.3126)$$
$$\boxed{Z = 108°}$$

Example 4: In ΔDEF, *d* = 8 m, *e* = 5 m, *f* = 10 m. Find the smallest angle in the triangle to the nearest tenth of a degree.

↳ is opposite the Smallest side!



$$e^2 = d^2 + f^2 - 2df\cos E$$

$$5^2 = 8^2 + 10^2 - 2(8)(10)\cos E$$

$$25 = 164 - 160\cos E$$

$$-164 \qquad -164$$

$$\frac{-139}{-160} = \frac{-160\boxed{\cos E}}{-160}$$

$$\cos E = 0.86875$$

$$E = \cos^{-1}(0.8675)$$

$$\boxed{E = 29.7^\circ}$$

[Answer: 29.7°]

Example 5: Two ships set sail from port P, heading in different directions. The first ship sails 7 km to R and the second ship sails 8 km to Q. If the distance between R and Q is 13 km, what is the angle between the directions of the two ships?

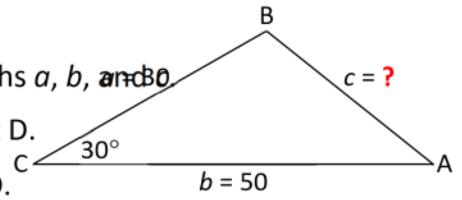
Assignment: Sec 2.4, p. 119 #Sec 2.4, p. 119 #1ac, 2ac, 4ad, 6, 12, 20 (which laws do you need in #20?!), opt: 10, 19.

DO NOT PRINT:

- Consider the oblique triangle ΔABC with side lengths a, b, and c.

  Draw an altitude of height h from vertex C to point D.

  There are now two right triangles, ΔACD and ΔBCD.



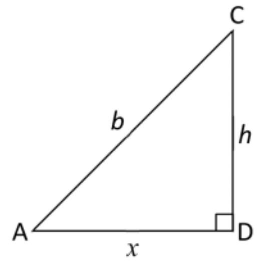
- Let x represent AD, then $c - x$ represents BD.

- In ΔACD

$$\cos A = \frac{x}{b}$$

$$b(\cos A) = x$$

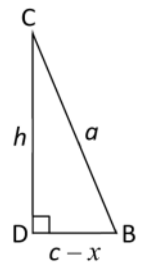
$$x = b\cos A$$



and the Pythagorean Theorem gives

$$b^2 = h^2 + x^2$$

- In ΔBCD The Pythagorean Theorem gives



$$a^2 = h^2 + (c - x)^2$$
$$a^2 = h^2 + c^2 - 2cx + x^2$$
$$a^2 = h^2 + x^2 + c^2 - 2cx$$
$$a^2 = b^2 + c^2 - 2cx$$
$$a^2 = b^2 + c^2 - 2c(b\cos A)$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Similarly $\quad b^2 = a^2 + c^2 - 2ac\cos B$

and $\quad c^2 = a^2 + b^2 - 2ab\cos C$

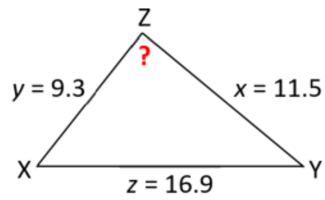
solution: In ΔABC, $\angle C = 30°$, $a = 30$ cm, and $b = 50$ cm.
Find c to the nearest hundredth of a centimetre.

- Draw a labelled diagram.
- $\angle C$ must be used, so the formula used is

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = (30)^2 + (50)^2 - 2(30)(50)\cos 30°$$
$$c^2 = 900 + 2500 - 3000\cos 30°$$
$$c^2 \approx 801.923\,789$$
$$c \approx 28.318\,259$$

Answer: $c \approx 28.32$ cm

solution: In ΔXYZ, $x = 11.5$, $y = 9.3$, and $z = 16.9$. Find the largest angle to the nearest degree.



- Draw a labelled diagram.
- The largest angle will be opposite the longest side. z is the longest side; $\angle Z$ is the largest angle.
- To find $\angle Z$ in ΔXYZ, the formula used must be

$$z^2 = x^2 + y^2 - 2xy\cos Z$$

$$\cos Z = \frac{x^2 + y^2 - z^2}{2xy}$$

$$Z = \cos^{-1}\left(\frac{x^2 + y^2 - z^2}{2xy}\right)$$

$$Z = \cos^{-1}\left(\frac{(11.5)^2 + (9.3)^2 - (16.9)^2}{2(11.5)(9.3)}\right)$$

$$Z = \cos^{-1}(-0.312\,622\,72)$$

$$Z \approx 108.217\,359^\circ$$

Answer: $\angle Z \approx 108^\circ$