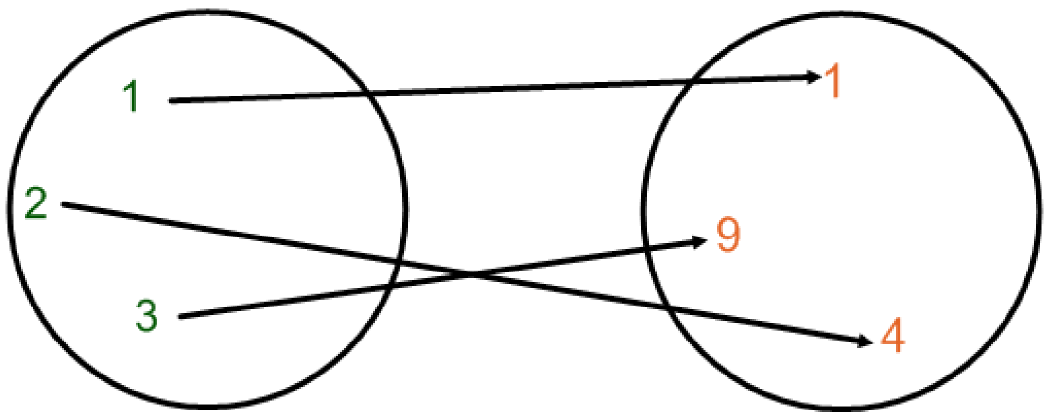
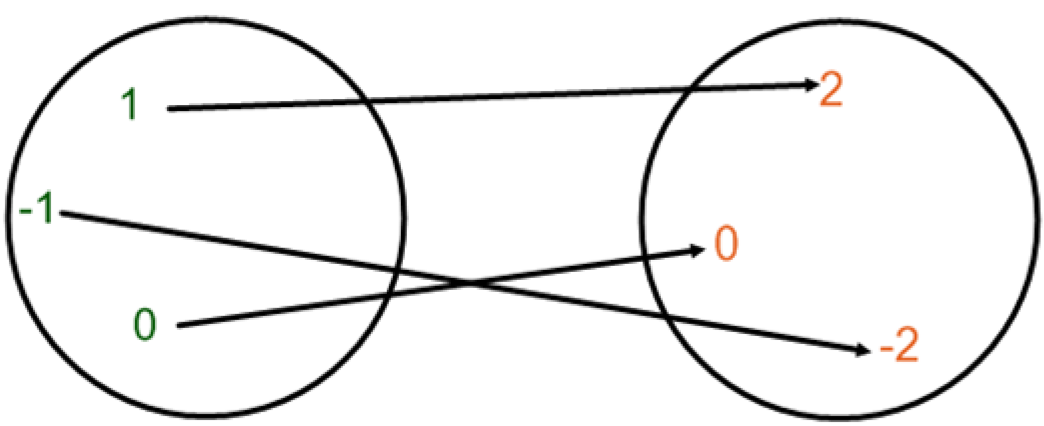
Inverse of a Relation

A relation is a mapping from one set onto another set.

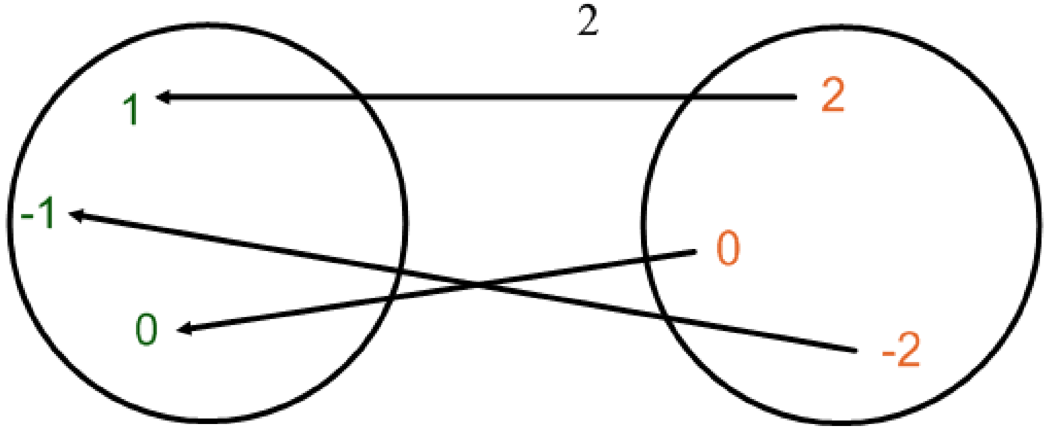


Let *A* and *B* be sets. The inverse of a relations $R: a \to b$ where $a \in A$ and $b \in B$ is a relation $R^{-1}: b \to a$ where $b \in B$ and $\in A$.

$$R : x \to 2x$$



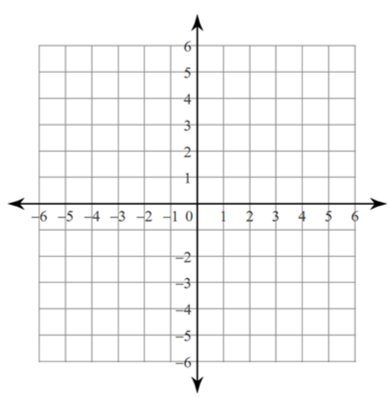
$$R^{-1} : x \rightarrow \frac{1}{2}x$$



Example 1

Graph the relation $y = 2x - 3$ and then graph the inverse of this relation.

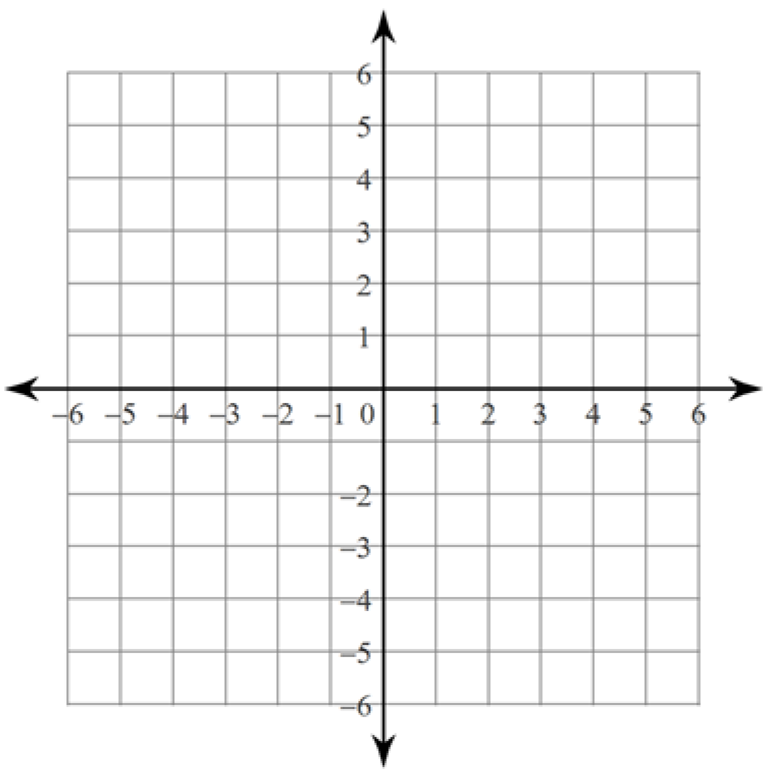
What do you notice?



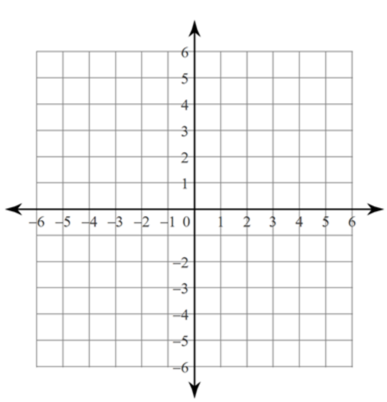
The graph of an inverse of a function $y = f(x)$ is a _______________ in the $y = x$ axis. In mapping notation this is: $(x, y) \rightarrow (y, x)$.

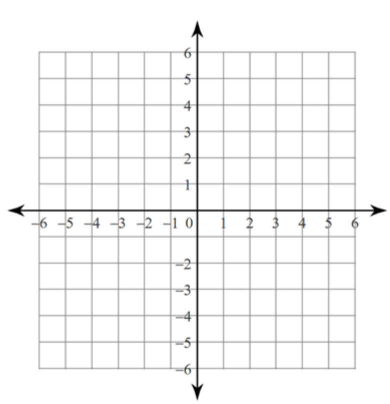
Example 2

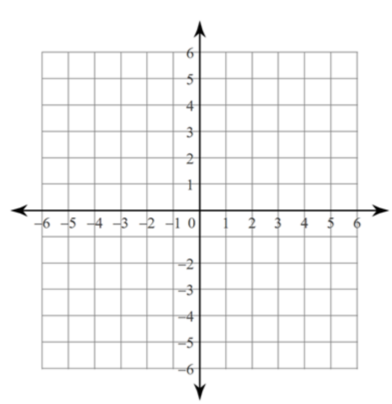
Graph the function $f(x) = x^2$ and its inverse on the same graph. State the domain and range the function and its inverse. Is the inverse a function? How can we restrict the domain of $f(x) = x^2$ so that its inverse is a function?



The ______________________ is a way to determine whether or not the inverse of a function, will also be a function.







When the inverse of a function $f(x)$ is itself a function, we may denote the inverse by $f^{-1}(x)$. Note that this is not an exponent. $f^{-1}(x) \neq \frac{1}{f(x)}$.

Example 3

Determine the equation of the inverse of $f(x) = \frac{1}{2}x - 1$.

Example 4

Given the function $f(x) = 4x - 3$ determine the value of $f^{-1}(2)$.